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Title:

Geometric aspects of the planar least gradient problem

Abstract:

The Dirichlet problem for the 1-Laplacian operator is the degenerate elliptic equation

$$-\mathrm{div}\left(\frac{Du}{|Du|}\right) = 0, \qquad u|_{\partial\Omega} = g \in L^1(\partial\Omega). \tag{1}$$

It is typically formulated in the variational form, as the *least gradient problem*

$$\min\left\{\int_\Omega |Du| : u \in BV(\Omega),\, u|_{\partial\Omega} = g \in L^1(\partial\Omega)\right\}. \tag{2}$$

In the talk, we will discuss geometric aspects of the least gradient problem, with the main focus on the planar case. In particular, equation (1) implies that the mean curvature of the level sets of a solution u vanishes. Hence, the 1-Laplace equation is linked to the study of minimal surfaces, and (in two dimensions) to optimal transport. We also present some applications of methods from geometric measure theory and optimal transport to existence, structure, and regularity of functions of least gradient (i.e., solutions to (2)).