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Title:

Conserved quantities and regularity in fluid mechanics

Abstract:

Evolutionary partial differential equations usually come endowed with certain additional, physically meaningful, quantities — such as energy. These quantities can be shown to be, at least formally, conserved, i.e., to depend only on the initial conditions and not change in time. However, it is notorious that such a formally derived conservation law may break down for weak solutions of low regularity. Indeed, not only do weak solutions defy uniqueness, they often posses the unmannerly trait of exhibiting nonphysical features, such as spontaneous creation or destruction of energy.

The intimate relation between regularity and conserved quantities is related to the concept of renormalisation in the celebrated DiPerna-Lions theory and the famous Onsager conjecture about energy conservation for the incompressible Euler equations. It states that there is a threshold regularity for conservation of kinetic energy — smooth enough weak solutions (namely those, which posses 1/3 of a spatial derivative) will always conserve energy, while below the threshold there is possibility of anomalous dissipation.

This research direction enjoyed an enormous amount of interest in the past few decades, and, famously, motivated the development of convex integration techniques in the context of fluid dynamics.

Both parts of the conjecture, the one guaranteeing energy conservation for smooth enough solutions, and the flexible one, became lively and potent research programmes, which culminated in the full proof of Onsager's prediction, as well as numerous studies for other systems of equations coming from fluid dynamics and general conservation laws. In my talk I will survey the most important developments in the context of compressible gas dynamics.