Finding a Puiseux expansion of a curve in parametric form

Segovia, YMIS 2010

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Question

Let us be given a curve in a parametric form

$$\begin{cases} x(t) & = t^4 + 2t^5 + 3t^6 + 4t^7 \\ y(t) & = t^6 + 3t^7 + 11t^8 + 30t^9 + 5t^{10}. \end{cases}$$

We ask what is the topological type of the singularity (Puiseux expansion)

$$y = x^{3/2} + c_1 x^{7/4} + c_2 x^{8/4} + \\ + c_3 x^{9/4} + c_4 x^{10/4} + c_5 x^{11/4} + \dots$$

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$$y = c_0 x^{3/2} + c_1 x^{7/4} + c_2 x^{8/4} + \\ + c_3 x^{9/4} + c_4 x^{10/4} + c_5 x^{11/4} + \dots$$

Standard approach

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$$x^{3/2} = (t^4 + 2t^5 + 3t^6 + 4t^7)^{3/2}$$

Therefore $y - x^{3/2}$ is equal to

$$5t^8 + 20t^9 + \frac{17}{8}t^{10} + \dots$$

Hence $c_1 = 0, c_2 = 5$. Now we look at $y - x^{3/2} - 5x^2$. After "simple" computations we get

$$y - x^{3/2} - 5x^2 = -\frac{417}{8}t^{10} - \frac{821}{8}t^{11} - \dots .$$

We get that $c_3 = 0$, $c_4 \neq 0$ and $c_5 \neq 0$ (in the latter we have to compute also $y - x^{3/2} - 5x^2 + \frac{417}{8}x^{5/2}$).

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$$x^{3/2} = t^6(1 + (2t + 3t^2 + 4t^3))^{3/2}$$

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$$x^{3/2} = t^6\left(1 + \frac{3}{2}(2t + 3t^2 + 4t^3) + \frac{3}{8}(2t + 3t^2 + 4t^3) + \dots\right)$$

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Look at the order at zero

Write

$$y = c_0 x^{q/p} + c_1 x^{(q+1)/p} + c_2 x^{(q+2)/p} + \dots.$$

We divide both sides by $x^{q/p}$. We get

where

$$P_1 = \dot{y}x - \frac{q}{p} y\dot{x}.$$

If the order of P_1 at zero is $q + (p-1) + r_1$, we know that $c_1 = \dots = c_{r_1 - 1} = 0 \neq c_{r_1}$. In the above example $P_1 = 10t^{11} + 65t^{12} + 35t^{13} - 85t^{14} - 165t^{15} - 10t^{16}$, so $r_1 = 2$, $c_1 = 0$ and $c_2 = 5$.

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$$y = c_0 x^{q/p} + c_1 x^{(q+1)/p} + c_2 x^{(q+2)/p} + \dots.$$

We divide both sides by $x^{q/p}$, differentiate with respect to t. We get

$$\frac{y}{x^{q/p}} = c_0 + c_1 x^{1/p} + c_2 x^{2/p} + \dots$$

$$\frac{\dot{y}x - \frac{q}{p} y \dot{x}}{x^{q/p+1}} = c_1 \frac{1}{p} \dot{x} x^{1/p-1} + c_2 \frac{2}{p} \dot{x} x^{2/p-1} + \dots$$

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Looking further at the order at zero

From the equation

$$P_1(t) = \frac{r_1 c_{r_1}}{p}\dot{x}x^{(q+r_1)/p} + \frac{(r_1+1)c_{r_1+1}}{p}\dot{x}x^{(q+r_1+1)/p} + \dots$$

we can go further dividing, differentiating and multiplying. We get

$$P_2 = S_2(r_2)c_{r_2}\dot{x}^3x^{(q+r_2)/p} + S_2(r_2+1)c_{r_2+1}\dot{x}^3x^{(q+r_2+1)/p} + \dots,$$

where

We see that again $c_{r_1+1} = \dots = c_{r_2-1} = 0 \neq c_{r_2}$. In our case

$$P_2 = -1680t^{19} - 11520t^{20} - 39060t^{21} - \dots.$$

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we assume here that ord $P_1 = q + (p-1) + r_1$ we can go further dividing, differentiating and multiplying. We get

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Hence $c_3 = 0$, $c_4 \neq 0$.

Looking further at the order at zero

We get

$$P_2 = S_2(r_2)c_{r_2}\dot{x}^3 x^{(q+r_2)/p} + \qquad S_2(r_2+1)c_{r_2+1}\dot{x}^3 x^{(q+r_2+1)/p} + \dots,$$

where

- $P_2 = x\dot{x}P_1' - (\frac{q+r_1}{p}\dot{x}^2 + x\ddot{x})P_1$.
- r_2 is such that $\operatorname{ord}_{t=0} P_2 = q + 3(p-1) + r_2$
- S_2 is some coefficient depending on q, p and r_1.

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General formula

In general from the expression

$$P_k = S_k(r_k)\dot{x}^{2k-1} c_{r_k} x^{(q+r_k)/p} + \dots$$

upon applying the above procedure we get

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$$P_{k+1} = x\dot{x}P_k' - P_k\left(\frac{q+r_k}{p}\dot{x}^2 + (2k+1)x\ddot{x}\right).$$

Then, if $r_{k+1} = \operatorname{ord} P_{k+1} - q - (2k+1)(p-1)$ we get that $c_{r_k+1} = \cdots = c_{r_{k+1}-1} = 0 \neq c_{r_{k+1}}$.

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Advantages

- Quick and elegant way to compute.
- We can also get the Puiseux coefficients, not only whether they are zero or no.
- Hard-core example: $x(t) = t^{12} + t^{13} + \frac{37}{28}t^{14}$, $y(t) = t^{18} + \frac{3}{2}t^{19} + \frac{33}{14}t^{20} + \frac{13}{14}t^{21} + \frac{675}{1568}t^{22} - \frac{675}{3136}t^{23}$.
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- If $a_p(s) \neq 0$ for $s \neq 0$, the Puiseux coefficients $c_j(s)$ are well-defined.
- If $a_p(0) = 0$, the limit $\lim_{s\to 0} c_j(s)$ might not exist, or loose its topological meaning.
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