Khovanov homology and periodic knots

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Periodic knots

Definition

A knot $K \subset S^3$ is $p$–periodic if it admits a rotational symmetry with the symmetry axis disjoint from $K$. 
Periodic knots

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Theorem (Murasugi criterion)

Suppose $K \subset S^3$ is a $p$-periodic knot with $p$ a prime. Let $\Delta$ be the Alexander polynomial of $K$ and $\Delta'$ be the Alexander polynomial of the quotient knot $K/\mathbb{Z}_p$. Let $l$ be the absolute value of linking number of $K$ with the symmetry axis. Then $\Delta_0 | \Delta$ and up to multiplication by a power of $t$ we have

$$\Delta \equiv \Delta_0^p (1 + t + \ldots + t^{l-1})^{p-1} \mod p. \quad (1)$$
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$$\Delta \equiv \Delta_0^p (1 + t + \ldots + t^{l-1})^{p-1} \mod p.$$  \hspace{1cm} (1)

Often one can find factors of $\Delta$ over integers.
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- Then $\mathbb{Z}_p$ acts on $\Sigma^m(K) - k$–fold branched cover;
- Then $\mathbb{Z}_p$ acts on $H_1(\Sigma^m(K))$. 

Look at $\mathbb{Z}_q^m$ summands of $H_1(\Sigma^m(K))$. 

$\mathbb{Z}_p$ can fix it, or permute different such summands. The number of fixed components is controlled by $|\Delta'(-1)|$. 

Restrictions for $H_1(\Sigma^m(K))$ of periodic knots.
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- These invariants appear with multiplicities.
- If $K$ is quasi-alternating, then $\Sigma^2(K)$ is an L–space and $d$-invariants are computable.
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Computable, if we know a representation.

It is known when knot group admits a representation into a dihedral group.

Other representations are sometimes harder to find.

None of the above criteria can be used for $\Delta = 1$ knots. The TAP criterion is possible, but requires finding non-trivial representations.
Theorem

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Jones and HOMFLYPT
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Theorem (HOMFLYPT criterion)

Let $\mathcal{R}$ be a unital subring in $\mathbb{Z}[a^{\pm 1}, z^{\pm 1}]$ generated by $a, a^{-1}, \frac{a + a^{-1}}{z}$ and $z$. For a prime number $p$ let $\mathcal{I}_p$ be the ideal in $\mathcal{R}$ generated by $p$ and $z^p$. If a knot $K$ is $p$-periodic and $P(a, z)$ is its HOMFLYPT polynomial, then

$$P(a, z) \equiv P(a^{-1}, z) \mod \mathcal{I}_p.$$
Theorem

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Przytycki shows an effective way of applying Theorem 4.
Equivariant Khovanov homology

For a periodic diagram $D$, the group $\mathbb{Z}_p$ permutes the cube of resolution.

Definition (Politarczyk)
For any $\Lambda$–module $M$ define the equivariant Khovanov homology as

$$EKh(L; M) = \text{Ext}_\Lambda(M, \text{CKh}(D; \mathbb{R}))$$

Does not depend on the choice of the diagram.

In a similar way one can show the existence of equivariant Lee theory.

Most important example: $M = \Lambda$. 

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- Most important example: $M = \Lambda$. 
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- We can define $\text{EKh}_d(L) = \text{EKh}(L; \mathbb{Z}[\xi_d])$ for any $d|p$. This is the third gradation, coming from representations of $\mathbb{Z}_p$.
- If $R = \mathbb{Z}_m$ and $p$ is invertible in $R$, then $\text{Ext}^i_{\Lambda} = 0$ for $i > 0$ and $\text{EKh}(L; \Lambda) = \text{Kh}(L; R)$. 
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- We can define $EKh_d(L) = EKh(L; \mathbb{Z}[\xi_d])$ for any $d|p$. This is the third gradation, coming from representations of $\mathbb{Z}_p$.
- If $R = \mathbb{Z}_m$ and $p$ is invertible in $R$, then $\text{Ext}^i_\Lambda = 0$ for $i > 0$ and $EKh(L; \Lambda) = Kh(L; R)$.
- On the other hand we have a Schur decomposition of $\text{Hom}_\Lambda(\Lambda; CKh(D))$. 
There exists an equivariant Lee spectral sequence (if $R = \mathbb{Z}_q$, $q > 2$, or $R = \mathbb{Z}$ or $R = \mathbb{Q}$).
Equivariant Lee spectral sequence

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The equivariant Khovanov polynomial and the equivariant Lee polynomial differ by a specific polynomial.
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Equivariant Lee homology for knots is easy.

The equivariant Khovanov polynomial and the equivariant Lee polynomial differ by a specific polynomial.

And $EKh(L; \Lambda)$ splits as a sum over different representations of $\Lambda$. 
Main criterion

Theorem (—, Politarczyk, 2017)

Let $K$ be a $p^n$-periodic, where $p$ is an odd prime. Suppose that $F = \mathbb{Q}$ or $F_r$, for a prime $r$ such that $r \neq p$, and $r$ has maximal order in the multiplicative group mod $p^n$. Set $c = 1$ if $F = F_2$ and $c = 2$ otherwise. Then the Khovanov polynomial $KhP(K; F)$ decomposes as

$$KhP(K; F) = P_0 + \sum_{j=1}^{n} (p^j - p^{j-1})P_j,$$

(2)

where

$$P_0, P_1, \ldots, P_n \in \mathbb{Z}[q^{\pm 1}, t^{\pm 1}],$$

are Laurent polynomials such that
1 \quad \mathcal{P}_0 = q^{s(K,F)}(q + q^{-1}) + \sum_{j=1}^{\infty} (1 + tq^{2cj})S_{0j}(t, q), \text{ and the polynomials } S_{0j} \text{ have non-negative coefficients;}
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1. $P_0 = q^{s(K,F)}(q + q^{-1}) + \sum_{j=1}^{\infty} (1 + tq^{2cj})S_{0j}(t, q)$, and the polynomials $S_{0j}$ have non-negative coefficients;

2. $P_k = \sum_{j=1}^{\infty} (1 + tq^{2cj})S_{kj}(t, q)$ and the polynomials $S_{kj}$ have non-negative coefficients for $1 \leq k \leq n$, 

If the width of $K_h(K,F)$ is equal to $w$, then $S_{kj} = 0$ for $j > c_2w$.

Without (4) the condition is trivial!
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4. \( P_k(-1, q) - P_{k+1}(-1, q) \equiv P_k(-1, q^{-1}) - P_{k+1}(-1, q^{-1}) \pmod{q^{p^{n-k}} - q^{-p^{n-k}}} \);
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We do not need to calculate equivariant Khovanov.
We work over \(\mathbb{Z}_3\).
Khovanov homology
Khovanov Polynomial

$$\text{KhP} = q + q^{-1} + (1 + tq^4)(t^{-7} q^{-15} + 3t^{-6} q^{-13} + t^{-5} q^{-11} +$$
$$+ 3t^{-4} q^{-9} + t^{-3} q^{-9} + 3t^{-2} q^{-7} + t^{-1} q^{-5} + 3t^{-1} q^{-3} + q^{-3} + q^{-1} + 3tq +$$
$$+ t^2 q^3 + 3t^3 q^3 + t^4 q^5 + 3t^5 q^7 + t^6 q^9 + 4(t^{-5} q^{-11} + t^{-4} q^{-9} + 2t^{-3} q^{-7} + 2t^{-2} q^{-5} +$$
$$+ t^{-1} q^{-5} + t^{-1} q^{-3} + 2tq^{-1} + q^{-3} + q^{-1} + 2t^2 q + t^3 q^3 + t^4 q^5)).$$
Tentative decomposition

\[ \text{KhP} = q + q^{-1} + (1 + tq^4)S'_01 + 4(1 + tq^4)S'_11, \]

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\[ S'_{11} = t^{-5} q^{-11} + t^{-4} q^{-9} + 2t^{-3} q^{-7} + 2t^{-2} q^{-5} + t^{-1} q^{-5} + t^{-1} q^{-3} + 2tq^{-1} + q^{-3} + q^{-1} + 2t^2 q + t^3 q^3 + t^4 q^5. \]
Main criterion

According to the main criterion define

$$\tilde{\Xi}(q) = (q + q - 1 + (1 + tq^4)(S'_0 - S'_1)) \mid t = -1$$

and set $$\Xi := (\tilde{\Xi}(q) - \tilde{\Xi}(q - 1)) \mod q^5 - q^7$$

We have then $$\Xi = -10q^5 + 5q^3 - 5q^7 + 10q^9$$

$$\Xi$$ is not zero, but we might get zero if we choose different $$S'_0$$ and $$S'_1$$.

In general checking all possibilities requires 20736 possibilities.
According to the main criterion define
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\( \Xi \) is not zero, but we might get zero if we choose different \( S'_{01} \) and \( S'_{11} \). In general checking all possibilities requires 20736 possibilities.
Speed up

Let $\delta = at^u q^i$. The change $S'_{11} \mapsto S'_{11} - \delta$, $S'_{01} \mapsto S'_{01} + 4\delta$ induces the change

$$\Xi \mapsto \Xi + aT_{ij},$$

where

$$T_{ij} = (-1)^i5(-q^{-j-4} + q^{-j} - q^{i} + q^{i+4}) \mod (q^5 - q^{-5}).$$
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$$T_{ij} = (-1)^i 5(-q^{-j-4} + q^{-j} - q^j + q^{j+4}) \mod (q^5 - q^{-5}).$$

Reducing modulo $q^5 - q^{-5}$ we get $T_{ij} = (-1)^i R_{j'}$ with $j' = j \mod 10$ and

$$R_1 = R_5 = 5(q - q^9),$$

$$R_3 = 10(q^3 - q^7),$$

$$R_7 = R_9 = 5(-q - q^3 + q^7 + q^9).$$
$\delta$ is such that $S_{11}' - \delta$ and $\delta$ have non-negative coefficients. Hence the question is, whether

$$\Xi = a_1 R_1 + a_3 R_3 + a_7 R_7$$

with

- $a_1 \in \{-1, 0, 1, 2, 3, 4, 5, 6\}$,
- $a_3 \in \{-3, -2, -1, 0\}$,
- $a_7 \in \{-4, -3, -2, -1, 0, 1, 2\}$.
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\begin{align*}
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  a_3 & \in \{-3, -2, -1, 0\}, \\
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\end{align*}
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This is impossible. The knot is not periodic.
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- Is $15n166130$ 5–periodic?