A HOPF GALOIS EXTENSION FROM THE QUANTUM SYMPLECTIC GROUP

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A notion which extends to noncommutative geometry the one of principal fiber bundles in differential geometry is the one of faithfully flat Hopf-Galois extensions. In the continuous limit, Hopf-Galois extensions are objects dual to principal fibrations.

In this talk I would like to give a concrete example of a noncommutative principal bundle $A(S^4_q) \subset A(S^7_q)$, a quantization of the Hopf bundle $S^7 \to S^4$, which is obtained starting form quantum groups.

The algebra $A(S^7_q)$, a deformation of the algebra of polynomials functions on S^7, is obtained as a subalgebra of the symplectic quantum group $Sp_q(2)$. This algebra turns out to be the "total space" of a quantum $SU_q(2)$-principal bundle over $A(S^4_q)$. The "base space" $A(S^4_q)$, a deformation of the algebra of polynomials on S^4, is the algebra generated by the entries of a projection $p \in Mat_4(A(S^7_q))$.

In particular, as said, it is possible to introduce a right coaction of $SU_q(2)$ on $A(S^7_q)$ such that the corresponding algebra of coinvariants coincides with $A(S^4_q)$ and the extension $A(S^4_q) \subset A(S^7_q)$ is Hopf-Galois and faithfully flat.

The projection p quantizes the anti-instanton of charge -1, here the topological charge is computed by comparing the class in K-theory of p with the fundamental K-homology class of $A(S^4_q)$.

Based on the paper "A Hopf bundle over a quantum four-sphere from the syplectic group" by Giovanni Landi, Chiara Pagani, Cesare Reina, [math.QA/0407342].