A DENSITY THEOREM FOR GLUINGS IN NONCOMMUTATIVE GEOMETRY

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We will first recall basic facts of the theory of C^*-algebras (the Gelfand-Naimark theorem and the GNS construction), which are fundamental for noncommutative geometry and give the motivation for considering noncommutative C^*-algebras as analogues of locally compact spaces. Then we will see how gluings correspond in the spirit of Gelfand-Naimark to pull-back diagrams in the commutative situation. Thus, pull-back constructions are the analogues of gluings in noncommutative geometry.

Concrete noncommutative spaces are in many cases constructed in two steps. In a first algebraic step one defines a $*$-algebra, for example in terms of generators and relations. In a second step one forms an enveloping C^*-algebra, making use of representations. In general, the procedures of gluing and of forming a C^*-closure are not compatible with each other, i.e., the gluing of two $*$-algebras is in general not dense in the gluing of their C^*-closures. The main point of the lecture will be to sketch the proof of a theorem, due to Matsumoto, giving a sufficient condition for compatibility. The proof uses factorial states and a Stone-Weierstraß theorem for such states, to be discussed on the way. Finally, several examples of gluings of noncommutative spaces, involving quantum discs, quantum spheres and noncommutative tori, will be considered.