

Selected topics in graph theory

15.10.2018 — homework, set 1

Problem 1. Prove that there exists a universal constant c such that for every $n \geq 3$ the Euler genus of the graph $K_{3,n}$ is between $n/2 - c$ and $n + c$.

Problem 2. Show that for every k there exists a 3-connected graph G_k with at most $1000k$ vertices such that G_k has at least $2^{k/1000}$ different cellular embeddings in a torus of facewidth 2. Here, we consider two embeddings different if their set of facial walks is different.

Problem 3. Let G be a graph and let Π be an embedding of G of Euler genus g . Assume that in this embedding there exists a facial walk W that contains a Π -onesided cycle. Prove that G can be embedded on a surface of Euler genus at most $g - 1$.