UNIVERSITY
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Statistical Data Analysis 2

Exercises # 1

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Problem 1: Conditional independence
Let X, Y, Z be random variables. X and Y are said to be *conditionally independent* given Z (in symbols $X \perp Y \mid Z$) if

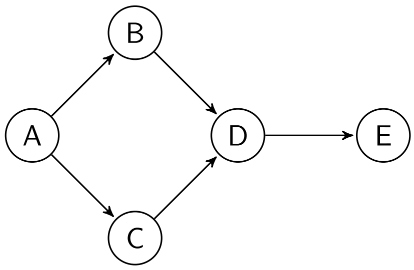
$$P(X, Y \mid Z) = P(X \mid Z)P(Y \mid Z).$$

This condition is equivalent to

$$P(X \mid Y, Z) = P(X \mid Z).$$

Using the laws of probability, show that this equivalence holds (show both directions of the proof).

Problem 2: Markov blanket
Consider the following graphical structure of a Bayesian network:

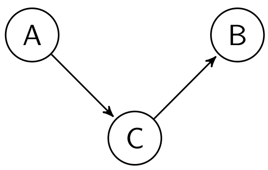


Determine the Markov blanket $\mathrm{MB}(C)$ of the node C and show that the conditional probability $P(C \mid A, B, D, E)$ can be expressed as

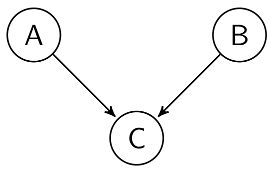
$$P(C \mid A, B, D, E) = P(C \mid \mathrm{MB}(C)).$$

Problem 3: Conditional independence and BNs
Consider the following graphical structures, corresponding to (different) Bayesian networks. For which network does the statement $A \perp B \mid C$ hold? For which does the statement $A \perp B$ hold? Prove your answers by the laws of probability.

a)



b)



Problem 4: Conjugate distributions
Let X be a binomially distributed random variable with parameters $N \in \mathbb{N}$ and $\theta \in [0, 1]$. Further,

assume that a prior $P(\theta)$ of the parameter θ is *beta distributed* with parameters α, β, i.e. its probability density function is given by

$$\rho(\theta) = \frac{\theta^{\alpha-1}(1-\theta)^{\beta-1}}{\int_0^1 t^{\alpha-1}(1-t)^{\beta-1}dt}$$

Show that the prior $P(\theta)$ is conjugate to the binomial likelihood $L(\theta) := P(X \mid \theta)$. In other words, show that the posterior distribution $P(\theta \mid X)$, which is defined as

$$P(\theta \mid X) = \frac{P(X \mid \theta)P(\theta)}{P(X)}$$

also obeys a beta distribution with suitable parameters.