Adaption makes it easy to integrate functions with unknown singularities

Leszek Plaskota[a], and Grzegorz W. Wasilkowski[b]

[a]Faculty of Mathematics, Informatics, and Mechanics, Warsaw University, Poland

[b]Department of Computer Science, University of Kentucky

We study numerical integration $I(f) = \int_0^T f(x)\,dx$ for functions f with *singularities*. Nonadaptive methods are inefficient in this case, and we show that the problem can be efficiently solved by *adaptive* quadratures at cost similar to that for functions with no singularities.

Consider first a class $\mathcal{F}_r$ of functions whose derivatives of order up to r are continuous and uniformly bounded for any but one singular point. We propose adaptive quadratures Q_n^*, each using at most n function values, whose *worst case* errors $\sup_{f\in\mathcal{F}_r} |I(f) - Q_n^*(f)| = \Theta(n^{-r})$. On the other hand, the worst case error of nonadaptive methods is $\Omega(n^{-1})$.

These worst case results do not extend to the case of functions with two or more singularities; however, adaption shows its power even for such functions in the *asymptotic setting*. That is, let F_r^∞ be the class of r-smooth functions with arbitrary (but finite) number of singularities. Then a generalization of Q_n^* yields adaptive quadratures Q_n^{**} such that $|I(f) - Q_n^{**}(f)| = O(n^{-r})$ for any $f \in F_r^\infty$. In addition, we show that for any sequence of nonadaptive methods there are 'many' functions in F_r^∞ for which the errors converge no faster than n^{-1}.

Results of numerical experiments are also presented.