Obtaining $O(N^{-2+\varepsilon})$ Convergence for Quadrature Rules Based on Digital Nets

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In this talk we investigate multivariate integration in reproducing kernel Sobolev spaces for which the second partial derivatives are square integrable. As quadrature points for our quasi-Monte Carlo algorithm we use digital (t, m, s)-nets over $\mathbb{Z}_2$ which are randomly digitally shifted and then folded using the baker's transformation. For this QMC algorithm we show that the root mean square worst-case error converges with order $2^{m(-2+\varepsilon)}$ for any $\varepsilon > 0$, where 2^m is the number of points. A similar result holds for lattice rules as shown by Hickernell.