A Lower Bound for the Sturm-Liouville Eigenvalue Problem on a Quantum Computer

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We study the complexity of approximating the lowest eigenvalue of a Sturm-Liouville differential equation on a quantum computer. This general problem includes the special case of computing the ground state energy of a quantum system for a given potential.

Recently Papageorgiou and Woźniakowski proved that quantum computers could achieve exponential speedups compared to classical computers for certain potentials. Let $\mathbb{L}_q$ be the differential operator for the Sturm-Liouville problem. Papageorgiou's and Woźniakowski's method uses the (discretized) unitary propagator $\exp(i\mathbb{L}_q)$ as a query. They showed that if the operator $\exp(ip\mathbb{L}_q)$ (a "power query") is computable in cost comparable to $\exp(i\mathbb{L}_q)$ for any integer p, one can solve the Sturm-Liouville problem with $\mathcal{O}(\log \epsilon^{-1})$ power queries.

In this paper we will prove a matching lower bound of $\Omega(\log \epsilon^{-1})$ power queries, therefore showing that $\Theta(\log \epsilon^{-1})$ power queries are sufficient and necessary. Our proof is based on a frequency analysis technique, which examines the probability distribution of the final state of a quantum algorithm and the dependence of its Fourier transform on the input.