People are Processors: Coalitional Auctions for Complex Projects

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ABSTRACT

To successfully complete a complex project, be it a construction of an airport or of a backbone IT system or crowd-sourced projects, agents (companies or individuals) must form a team (a coalition) having required competences and resources. A team can be formed either by the project issuer based on individual agents' offers (centralized formation); or by the agents themselves (decentralized formation) bidding for a project as a consortium—in that case many feasible teams compete for the employment contract. In these models, we investigate rational strategies of the agents (what salary should they ask? with whom should they team up?) under different organizations of the market. We propose various concepts allowing to characterize the stability of the winning teams. We show that there may be no (rigorously) strongly winning coalition, but the weakly winning and the auction-winning coalitions are guaranteed to exist. In a general setting, with an oracle that decides whether a coalition is feasible, we show how to find winning coalitions with a polynomial number of calls to the oracle. We also determine the complexity of the problem in a special case in which a project is a set of independent tasks. Each task must be processed by a single agent, but processing speeds differ between agents and tasks.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multiagent systems*

Keywords

game theory, co-opetition, cooperative game theory, coalition formation, equilibria, skill games, scheduling

1. INTRODUCTION

Crowd-sourcing and open collaboration systems have become popular over the last decade [1], and frequently provide cost-effective on-demand human resources to carry out a range of tasks, from simple ones like annotating and disambiguating snippets of text, to creation of complex software and knowledge repositories.

Oftentimes, isolated simple micro-tasks are outsourced to individuals, and there is no explicit notion of team-work among them. In other instances, e.g., collaborative creation of knowledge or software repositories like Wikipedia and open-source softwares, sustained interaction among a small set of skilled knowledge workers is involved. We are interested in the latter scenarios, that of complex projects that typically require multiple skills and coordination.

However, in contrast to the currently predominant usage of crowd-sourcing, where contributions are often best effort, we envision that with the proliferation of maturing crowd-sourcing and collaboration platforms, we will witness in the immediate future the use of crowd-sourcing in more critical projects, where the owner of a task may want to achieve better guarantees of delivery of a complete job, including the integration of the various parts, rather than out-source the tasks in a piece-meal manner, and likewise, the stakes for the contributors will be higher as well, so that they will compete for higher payoffs, even while collaborating.

This paper abstracts such a market-place, where a pool of knowledge workers with specific skill sets will try to bid for complex tasks, possibly individually, or alternatively as composite teams, where individuals aim to maximize their revenues - which is affected, depending on whether they manage to attract any tasks at all, as well as the way the rewards for a complex job are split among the team members, while the clients would typically aim for timely completion of their projects in a cost-effective (e.g., based on a predetermined budget) manner. While motivations of our abstract model stem from the crowd-sourcing scenarios, it fits well in other real world scenarios, for instance when consortiums are formed to bid for a tender of a complex project, and individual organizations may not have the resources and capacity to fulfill all the needs of the project, and yet benefit from being part of a coalition, likewise, for the client proposing the project would/can not distribute the tasks in a piece-meal manner, and will instead like a single logical entity (the consortium) to deal with the project as a whole.

The contributions of this paper are as follows: (i) First we identify and formalize a new class of coalition games. These games describe the agents gathering into groups and competing with other teams for the employment in a complex project. In a general setting we consider an oracle that decides which teams of agents (further referred to as coalitions) have sufficient skills to complete the project on time. In this way our games resemble cooperative skill games [2] and coalitional resource games [3] (these games, however, consider the problems in the grand coalition and interaction between its members; our approach is to expose multiple coalitions' competition). Thus, we do not apply the typical cooperative game theory concepts [4], and instead model the cooperation and the competition of the agents as the non-cooperative games. Our model also generalizes the coalition formation problems to the strategic agents (thus, the algorithms for coalition formation [5] can be used in our setting as the general mechanism to solve the subproblem of finding (cheapest/best) feasible coalitions).

Next we explore two organizations of the market. In the *centralized setting* (Section 3), where the agents communicate only with

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the client, (ii) we prove that a Strong Nash Equilibrium (SNE) always exists unless there is no feasible coalition. We show how to find SNE, and for the client—how to select the best coalition, with a polynomial number of calls to the orcale. In a *decentralized setting* (Section 4) (iii) we show two concepts of winning coalitions. We prove that a strongly winning coalition may not exist, but a weakly winning coalition is guaranteed to exist (provided there exists a feasible one). We show how to find weakly/strongly winning coalitions. In Section 5 (iv) we propose two mechanisms that the client can apply to find the winning team. We introduce the concept of an auction-winning coalition and show how to find one (this relates to the works on all-pay auctions in the context of crowdssourcing [6]). In Section 6 we propose how the general oracle can be replaced with a concrete scheduling model and (v) determine the exact complexity of the problem.

Mapping the abstract model to real applications, including incorporation of trust, social relations, skill and historical performance information, etc., are out of the scope of this paper, and we leave these out as avenues to extend the current work in future.

2. THE GENERAL MODEL

We consider a model in which a client (an issuer) submits a single complex project to be executed. The project has a deadline d. The client has a certain valuation v of the project, that is the maximal price that she is able to pay for completing the project. The client has no additional utility from completing the project before the deadline: if she had, it could be expressed by changing the project description and submitting a project with shorter deadline.

There is a set $N = \{1, 2, \dots n\}$ of n agents. For each agent i we define $\phi_i^{\min} > 0$ to be the agent's *minimal salary* for which i is willing to work. The agent prefers to work for $\phi_i^{\min}$ than not to work (and then to work for higher salary). The value $\phi_i^{\min}$ is private to the agent—neither the issuer nor the other agents know $\phi_i^{\min}$. However, in order to construct equilibria of a game, an agent has to have some beliefs about the minimal salaries of other agents.

A subset of the agent's population N forms a coalition (a team) to be awarded the project; the paper's core contribution is on how this process should be organized.

A coalition $\mathcal{C}$ is a triple $\langle N_\mathcal{C}, \phi_\mathcal{C}, c_\mathcal{C} \rangle$ consisting of the set of participating agents $N_\mathcal{C} \subseteq N$, a salary function $\phi_\mathcal{C} : N_\mathcal{C} \to \mathbb{N}$ assigning salaries to member agents, and the total cost of the coalition $c_\mathcal{C} \in \mathbb{N}$—the total amount of money earned by the participants of $\mathcal{C}$. Salaries are discrete to represent some minimal reasonable changes in the agents' compensation (not only money is discrete, but also it is common in real-world auctions to specify a minimal difference between two successive bids).

For a coalition, there is a *schedule* that for each agent i specifies t_i, the time that i needs to spend working on the project; we give an example of the concrete schedule definition in Section 6)

We consider two models of agents' compensation. Let $\phi_\mathcal{C}^{tot}(i)$ denote the total amount of money agent i gets in coalition $\mathcal{C}$ (naturally, $c_\mathcal{C} = \sum_{i \in N_\mathcal{C}} \phi_\mathcal{C}^{tot}(i)$). In the *project salary* model $\phi_\mathcal{C}^{tot}(i)$ is equal to the salary of the agent $\phi_\mathcal{C}(i)$ (and thus does not depend on the amount of work assigned to that agent). In the *hourly salary* model $\phi_\mathcal{C}^{tot}(i)$ is equal to the product of the salary $\phi_\mathcal{C}(i)$ and the time t_i that i spends on processing her part of the project (t_i is known from the schedule).

In the project salary model the agents are interested in earning as much money as possible. The hourly salary model represents a different environment in which agents perhaps work on many projects simultaneously; thus the agents are interested in having maximal salary per time unit (e.g. an agent prefers to work $t_i = 1$ time unit with a salary $\phi_i = 3$ to working $t_i = 2$ time units for $\phi_i = 2$).

The coalition $\mathcal{C}$ is *feasible* if there exist a schedule such that: (i) the project can be finished before the deadline (if the coalition does not have some required competences, we model this as a coalition that never finishes a project); (ii) the project budget is not exceeded ($c_\mathcal{C} \leq v$); (iii) the cost $c_\mathcal{C}$ of the coalition $\mathcal{C}$ is consistent with the salaries $\phi_\mathcal{C}$. Specifically, in the project salary model $c_\mathcal{C} = \sum_{i \in N_\mathcal{C}} \phi_\mathcal{C}(i)$. In the hourly salary model there must exist a schedule in which each member i of the coalition $\mathcal{C}$ spends t_i time units on the project and $c_\mathcal{C} = \sum_{i \in N_\mathcal{C}} t_i \phi_\mathcal{C}(i)$. Moreover, the salaries are higher than the minimal salaries, $\phi_\mathcal{C}(i) \geq \phi_i^{\min}$.

A coalition $\mathcal{C}$ is *cheaper* than $\mathcal{C}'$ if it has a strictly lower cost $c_\mathcal{C} < c_{\mathcal{C}'}$ or if it has the same cost, but it is preferred by a deterministic tie-breaking rule $\prec$, $N_\mathcal{C} \prec N_{\mathcal{C}'}$. A tie-breaking rule may take into account non-economical preferences that are considered to be second-important to the cost of the coalitions. Using a specific tie-breaking rule influences the complexity of the problem of finding the most-preferred coalition. We use a lexicographic tie-breaking rule (where a coalition is represented by the concatenation of the sorted list of the names of its members), however the results can be generalized to any other deterministic rule with the only difference of the complexity of FCFC (described below).

Throughout the paper we use two following problems: FIND FEASIBLE COALITION (FFC) and FIND CHEAPEST FEASIBLE COALITION (FCFC). We reduce other problems to FFC and FCFC (we will also show that FCFC can be polynomially reduced to FFC).

PROBLEM 1. (FFC: FIND FEASIBLE COALITION). *An instance of FFC consists of a project (with a deadline d and a budget v) and the set of the agents N with (known) minimal required salaries $\phi_i^{\min}$. The question is to find any feasible coalition or to claim there is no such.*

PROBLEM 2. (FCFC: FIND CHEAPEST FEASIBLE COALITION). *An instance is the same as in the* FFC *problem. The question is to find the cheapest feasible coalition or to claim there is no such.*

We use the general model as defined above in Sections 3, 4, and 5. In these sections, we assume that there is an oracle solving FFC. This allows us to study a very general setting of the problem, abstracting from concrete notions of a schedule or a division of labor in a coalition. To solve FFC, the oracle must know the underlying model, in particular the minimal salaries of agents and the maximal price of the project v; we may also assume that the oracle is local to an agent—the oracle knows the minimal salary of this agent and agent's beliefs about minimal salaries of others. Then, to get the exact computational results, we need to define in a compact form, e.g. which coalitions are able to complete the project before the deadline. In Section 6 we consider a specific model of FFC in which the project is a set of independent, indivisible tasks and the agents have certain skills, i.e., speeds with which they process the tasks (a lack of skills is modeled as zero speed).

We consider two models of forming coalitions. First, in Section 3, we consider the *centralized formation*. Agents submit their bids—(asking) salaries ϕ_i—directly to the client (project issuer). The client chooses the members of the team that is awarded the project (we will call the winning team the coalition to use the same vocabulary as in the second part of the paper). Naturally, the client chooses the members so that the project is completed before the deadline for the smallest price. The members of the winning coalition are payed according to their asking salaries ϕ_i.

Second, in Sections 4 and 5, we consider the *decentralized formation* of the coalition. Agents communicate and are able to form coalitions by binding agreements. A coalition sends a bid—the total cost $c_{\mathcal{C}}$—to the client; the bid represents the compensation the coalition expects to get for completing the whole project. The cheapest coalition $\mathcal{C}^*$ wins the project and is payed $c_{\mathcal{C}^*}$; then $c_{\mathcal{C}^*}$ is allotted to the members of $\mathcal{C}^*$ according to the salary function $\phi_{\mathcal{C}^*}$.

3. CENTRALIZED FORMATION

In the centralized model we assume that the agents submit their asking salaries ϕ_i directly to the client (the issuer of the project). The client, having the asking salaries of the agents, wants to form the cheapest feasible coalition (that is able to complete the project before the deadline). In this section, we first show that this problem reduces to FFC, the problem of finding a feasible coalition. Then, we analyze the optimal bidding strategies of agents.

PROPOSITION 1. *The problem* FCFC *can be solved in time* $O((\log v + n)$*ffc*), *where ffc is the complexity of* FFC.

PROOF. Some proofs are ommitted due to space constraint, and are available in an anonymized full version at `www.dropbox.com/s/74la1mz03x1m9x9/crowdsourcing.pdf` □

PROPOSITION 2. *Having the asking salaries of the agents, the problem of finding the winning team can be solved in time* $O($*fcfc*$)$, *where fcfc is the complexity of* FCFC.

PROOF. One needs to solve FCFC with the minimal salaries of the agents set to their asking salaries ($\phi_i^{\min} = \phi_i$). □

The agents may be strategic and manipulate their asking salaries to maximize their payoffs. This problem is a strategic game. An action of the agent i is her asking salary $\phi_i \geq \phi_i^{\min}$. The payoff of i is ϕ_i if and only if i is a member of the cheapest feasible coalition; otherwise it is equal to 0.

Interestingly, in such setting, in the project salary model there exist sets which are stable against collaborative actions of the agents. We recall that a vector of the agents' actions is a Strong Nash Equilibrium (SNE, [7]) if no subset of the agents can change its actions so that all the deviating agents would obtain strictly better payoffs.

For each subset of the agents $N' \subseteq N$, by $\mathcal{C}^*(N')$ we denote the cheapest feasible coalition using only the agents from N' (the coalition $\mathcal{C}^*(N')$ does not exist if there is no feasible coalition consisting of the agents from N').

THEOREM 3. *In the project salary model, if there exists a feasible coalition then there exists a Strong Nash Equilibrium. In every SNE, the set of the agents that get positive payoffs is the set of agents forming the cheapest feasible coalition,* $N_{\mathcal{C}^*(N)}$.

PROOF. Let $N^* = N_{\mathcal{C}^*(N)}$ be the set of the agents participating in the cheapest feasible coalition. We say that the action ϕ_i of the agent i is *minimal* if and only if $\phi_i = \phi_i^{\min}$. We show how to construct the asking salaries ϕ_i^* of the agents from N^* that, together with the minimal actions of the agents outside N^*, form the Strong Nash Equilibrium. A sketch of proof is as follows. We show the set of linear inequalities for the variables $\phi_i, i \in N^*$. Let us denote the maximal values of ϕ_i which satisfy the inequalities as ϕ_i^* (maximal in the sense that if we increase any value ϕ_i^*, then the new values will not satisfy all the inequalities any more). We show that the actions ϕ_i^* of the agents from N^*, together with the minimal actions of the agents outside of N^*, form an SNE and that the set of the solutions ϕ_i^* that satisfy all the inequalities is nonempty.

The first inequality states that the values ϕ_i must lead to a feasible solution: $\sum_{i \in N^*} \phi_i \leq v$. Next, as $\mathcal{C}^*$ is the cheapest feasible coalition, for each feasible coalition $\mathcal{C}'$ ($N^* \neq N_{\mathcal{C}'}$) such that $N^* \prec N_{\mathcal{C}'}$, $\mathcal{C}^*$ must have (weakly) lower cost: $\sum_{i \in N^* \setminus N_{\mathcal{C}'}} \phi_i \leq \sum_{i \in N_{\mathcal{C}'} \setminus N^*} \phi_i^{\min}$. For $\mathcal{C}'$ preferred over $\mathcal{C}^*$ ($N^* \neq N_{\mathcal{C}'}$ and $N_{\mathcal{C}'} \prec N^*$), $\mathcal{C}^*$ must have strongly lower cost: $\sum_{i \in N^* \setminus N_{\mathcal{C}'}} \phi_i < \sum_{i \in N_{\mathcal{C}'} \setminus N^*} \phi_i^{\min}$.

First, if the values ϕ_i^* satisfy the above inequalities and the agents outside of N^* play their minimal actions, then the agents from N^* will get positive payoffs. If they did not get the positive payoffs, it would mean that there exists a feasible cheaper coalition $\mathcal{C}'$. However, the inequalities ensure that the agents from $N^* \setminus N_{\mathcal{C}'}$ induce the lower total cost than the total cost of the agents from $N_{\mathcal{C}'} \setminus N^*$; this ensures that agents N^* with actions ϕ_i^* form a cheaper coalition than $\mathcal{C}'$.

Next, we show that no set of agents $N_{\mathcal{C}'}$ can make a collaborative action $\overline{\phi}$ after which the payoff of all agents from $N_{\mathcal{C}'}$ will be greater than previously. For the sake of contradiction let us assume that there exists such a set of agents $N_{\mathcal{C}'}$ and such an action $\overline{\phi}$. First we consider the case when the payoff of some agent $i \notin N^*$ would change. This means that after $\overline{\phi}$ there would be a new cheapest feasible coalition $\mathcal{C}'$, where $i \in N_{\mathcal{C}'}$. However, we know that the total cost of the agents from $N^* \setminus N_{\mathcal{C}'}$ is lower than the total cost of the agents from $N_{\mathcal{C}'} \setminus N^*$. This means that $\mathcal{C}'$ cannot be cheaper than the coalition consisting of the agents from N^*. Finally, consider the case when only the payoffs of the agents from N^* change (and thus $N_{\mathcal{C}'} \subseteq N^*$). However, if the strict subset of N^* could complete the project before the deadline, then $\mathcal{C}^*(N)$ would not be the cheapest. Thus, $N_{\mathcal{C}'} = N^*$. This means that every agent from N^* must have played a higher action (and others must have not changed their actions). Since ϕ_i^* were maximal, after the action $\overline{\phi}$ some inequality, for some feasible coalition $\mathcal{C}''$, would not hold any more. Thus, we infer that $\mathcal{C}''$ is cheaper than $\mathcal{C}'$.

To check that there always exists a solution, we see that the values $\phi_i^* = \phi_i^{\min}$ satisfy all inequalities.

Finally, by contradiction we prove the N^* is formed by the same agents as forming the cheapest feasible coalition. Assume that the set of the agents that get positive payoffs in some SNE is $N' \neq N^*$. However, if the agents from $(N^* \setminus N')$ play their minimal actions, then the coalition consisting of the agents from N^* would be cheaper than the coalition consisting of the agents from N' (and so, it would be feasible). Thus, the agents from $(N^* \setminus N')$ can deviate, getting better payoffs. This completes the proof. □

There is no analogous result for hourly salary model.

PROPOSITION 4. *In the hourly salary model there may not exist a Strong Nash Equilibrium even though there exists a feasible coalition.*

The proof of Theorem 3 is constructive, however it requires considering all feasible coalitions, and so, leads to the potentially high complexity. On the other hand, if the salaries of the agents can be rational numbers, we can find the salary function in SNE by a polynomial reduction to the FCFC problem. This result is particularly interesting if the salaries of the agents have high granularity; rounding such a rational solution gives the integral solution which might not be exactly correct, but the error has a small magnitude.

PROPOSITION 5. *In the project salary model, if the salaries of the agents are rational, then finding a Strong Nash Equilibrium can be solved in time* $O(n^3 \log(nv)$*fcfc*$)$, *where fcfc is the complexity of* FCFC.

PROOF. First, we solve a single instance of the FCFC problem to find $N^* = N_{\mathcal{C}^*(N)}$. Next, similarly as in the proof of Theorem 3, we introduce the variables $\phi_i, i \in N^*$ and the inequalities (also the same as in the proof of Theorem 3). If we find the values $\phi_i, i \in N^*$ satisfying all the inequalities, then the values $\phi_i, i \in N^*$, together with the minimal salaries of the agents outside of N^*, will form a Strong Nash Equilibrium.

The set of inequalities given in the proof of Theorem 3 is a linear program; there are, however, exponentially many constraints (a constraint for each possible coalition). We construct a separation oracle by a polynomial reduction to the FCFC problem. Since ellipsoid method [8] requires $O(n^3 L)$ calls to the separation oracle [9] (where L is the size of the representation of the problem; here $L = O(\log(nv))$), this allows us to solve the linear program in time $O(n^3 \log(nv) fcfc)$.

To check whether all the inequalities are satisfied, it is sufficient to solve FCFC with the following parameters. The minimal salaries of the agents from N^* are set to the values of the variables ϕ_i ($\forall_{i \in N^*} \phi_i^{\min} := \phi_i$). The minimal salaries of the agents outside of N^* are left unmodified. Let $\mathcal{C}$ denote the solution of such instance of the FCFC problem. There exists a not-satisfied inequality if and only if $N_\mathcal{C} \neq N^*$. The not-satisfied inequality is the inequality that corresponds to the coalition $\mathcal{C} \neq \mathcal{C}^*$. This completes the proof. $\square$

PROPOSITION 6. *Checking whether a given vector of the asking salaries $\langle \phi_i \rangle, i \in N$ is a Strong Nash Equilibrium can be solved in time O(fcfc), where fcfc is the complexity of* FCFC.

4. DECENTRALIZED FORMATION

Assume that the agents can communicate and agree their strategies. Consequently, they can form coalitions and bid for the project as consortiums. We show the concept of a (rigorously) strongly winning coalition, where no subset of agents can successfully deviate. We show how to characterize (rigorously) strongly winning coalitions and how to reduce the problem of finding them to the FCFC problem. We show that the strongly winning coalitions may not exist, and so we introduce the concept of a weakly winning coalition. We prove that a weakly winning coalition always exist and demonstrate how to reduce the problem of finding them to the FCFC problem.

We model the behavior of the agents as a game. Agent i's action is a triple $\langle N_\mathcal{C}, \phi_\mathcal{C}, b_\mathcal{C} \rangle$. Intuitively, such an action means that the agent i decides to enter the coalition $\mathcal{C} = \langle N_\mathcal{C}, \phi_\mathcal{C}, b_\mathcal{C} \rangle$. The payoff of the agent is equal to $\phi_\mathcal{C}(i)$ if (i) $\mathcal{C}$ is feasible, (ii) each agent $j \in N_\mathcal{C}$ agrees to participate in $\mathcal{C}$ (i.e. they all play $\mathcal{C}$), and (iii) there is no feasible cheaper coalition $\mathcal{C}'$ such that all the agents from $N_{\mathcal{C}'}$ agree to participate in $\mathcal{C}'$. Otherwise, the payoff of i is 0.

4.1 Strongly winning coalitions

In this game the payoffs depend on whether the others agree to cooperate, thus the Strong Nash Equilibrium (SNE) rather than the Nash Equilibrium [10] should be considered. In the following definition we propose an even more stable equilibrium concept than the SNE—the Rigorously Strong Nash Equilibrium (RSNE). The RSNE requires that no subset of agents can deviate so that each would get a payoff *at least as good* (instead of strictly better). Our approach is motivated by considering a risk-averse agents. In a SNE, the agents have no incentive to deviate if they get the same payoff; however they also have no incentive not to deviate. Yet, any deviation will result in a serious payoff loss for some agents (changing their payoffs from a positive ϕ to zero). A risk-averse agent will prefer not to be exposed to the possibility of such loss.

DEFINITION 1. *The vector of actions π is a* Rigorously Strong Nash Equilibrium (RSNE) *if and only if there is no subset of the agents $N_\mathcal{C}$ such that the agents from $N_\mathcal{C}$ can make a collaborative action $\mathcal{C}$ (a set of actions played by agents) after which the payoff of each agent i from $N_\mathcal{C}$ would be at least equal to her payoff under π and the payoff of at least one agent $i \in N$ would change.*

In the above definition the requirement that the payoff of at least one agent $i \in N$ must change after the coalition deviates ensures that we treat as equivalent the coalitions with the same payoffs. For instance, assuming a system with three agents, a, b and c, if the coalition $\{a, b\}$ gets a positive payoff, it does not matter whether c plays $\langle \{c\}, v+1 \rangle$ or $\langle \emptyset, v+1 \rangle$: in both cases all payoffs are the same.

Below we introduce additional definitions that help to characterize the RSNE in our game.

DEFINITION 2. *A feasible coalition $\mathcal{C}$ is* explicitly endangered *by a coalition $\mathcal{C}'$ if (i) $\mathcal{C}'$ is feasible, (ii) $N_\mathcal{C} \cap N_{\mathcal{C}'} = \emptyset$ and (iii) $\mathcal{C}'$ is cheaper than $\mathcal{C}$.*

A feasible coalition $\mathcal{C}$ is implicitly endangered *by a coalition $\mathcal{C}'$ if (i) $\mathcal{C}'$ is feasible, (ii) $N_\mathcal{C} \cap N_{\mathcal{C}'} \neq \emptyset$ and each agent from $N_\mathcal{C} \cap N_{\mathcal{C}'}$ gets in $\mathcal{C}'$ at least as good salary as in $\mathcal{C}$, and (iii) either $N_\mathcal{C} \neq N_{\mathcal{C}'}$ or $\phi_\mathcal{C} \neq \phi_{\mathcal{C}'}$.*

If there are agents belonging to both coalitions ($N_\mathcal{C} \cap N_{\mathcal{C}'} \neq \emptyset$), we do not consider the total cost of the alternative coalition $\mathcal{C}'$, as the decision whether $\mathcal{C}'$ will be formed depends solely on the agents from $N_\mathcal{C} \cap N_{\mathcal{C}'}$: if they decide to form $\mathcal{C}'$, $\mathcal{C}$ will not be formed, thus the client won't be able to choose between $\mathcal{C}$ and $\mathcal{C}'$.

Informally, a coalition is (rigorously) strongly winning if it constitutes a (rigorous) Strong Nash Equilibrium, i.e., the members will not deviate to other coalitions.

DEFINITION 3. *The feasible coalition $\mathcal{C}$ is* rigorously strongly winning *if and only if there is an RSNE in which the agents from $N_\mathcal{C}$ get positive payoffs $\phi_\mathcal{C}$.*

DEFINITION 4. *The feasible coalition $\mathcal{C}$ is* strongly winning *if and only if there is an SNE in which the agents from $N_\mathcal{C}$ get positive payoffs $\phi_\mathcal{C}$.*

The following theorem connects the intuitive notion of endangerment with the notion of a winning coalition.

THEOREM 7. *The coalition $\mathcal{C}$ is rigorously strongly winning if and only if $\mathcal{C}$ is not explicitly nor implicitly endangered by any coalition.*

PROOF. $\Longleftarrow$ Assume that there exists a rigorously strongly winning coalition $\mathcal{C}$; thus there exists a Rigorously Strong Nash Equilibrium $RSNE$ in which the agents from $N_\mathcal{C}$ get positive payoffs. This implies that the agents from $N_\mathcal{C}$ agree on the action $\langle N_\mathcal{C}, \phi_\mathcal{C}, b_\mathcal{C} \rangle$; other agents ($N \setminus N_\mathcal{C}$) have zero payoffs. For the sake of contradiction let us assume that there exists a feasible coalition $\mathcal{C}'$ such that $\mathcal{C}$ is explicitly or implicitly endangered by $\mathcal{C}'$.

If $N_\mathcal{C} \cap N_{\mathcal{C}'}$ is empty ($\mathcal{C}$ is explicitly endangered by $\mathcal{C}'$), then $N_{\mathcal{C}'}$ must be cheaper. This however contradicts the assumption that the agents from $N_\mathcal{C}$ get positive payoffs.

Assume thus that $N_\mathcal{C} \cap N_{\mathcal{C}'}$ is non-empty (i.e., $\mathcal{C}$ is implicitly endangered by $\mathcal{C}'$). Consider the following collaborative action of agents $(N \setminus N_\mathcal{C}) \cup N_{\mathcal{C}'}$. All the agents from $N_{\mathcal{C}'}$ make action $\mathcal{C}'$. Each agent i from $N \setminus (N_\mathcal{C} \cup N_{\mathcal{C}'})$ makes an action $\langle \{\}, \phi_\emptyset \rangle$, where $\phi_\emptyset$ is an empty function. We show that after playing this action no agent from $(N \setminus N_\mathcal{C}) \cup N_{\mathcal{C}'}$ will get lower payoff and that some agents will get a strictly better payoff (which will contradict the assumption that $RSNE$ is a Rigorously Strong Nash Equilibrium). Clearly each agent from $N \setminus (N_\mathcal{C} \cup N_{\mathcal{C}'})$ does not decrease her payoff (as previously it was equal to 0). Now, we show that the agents from $N_{\mathcal{C}'}$ will get at least the same payoff as before. Since we know that $\mathcal{C}$ is implicitly endangered by $\mathcal{C}'$ (and thus the agents from $N_{\mathcal{C}} \cap N_{\mathcal{C}'}$ get in $\mathcal{C}'$ at least as good payoff as in $\mathcal{C}$) it is sufficient to show that the agents from $N_{\mathcal{C}'}$ will get positive payoffs. Indeed, there is no feasible coalition that includes some agents from $N \setminus (N_{\mathcal{C}} \cup N_{\mathcal{C}'})$ (as these agents play $\{\}$). Also, the agents from $N_{\mathcal{C}} \setminus N_{\mathcal{C}'}$ do not agree on the collaborative action (they still play $\mathcal{C}$) and thus, cannot form a feasible coalition. Thus, after such change of played actions $\mathcal{C}'$ is the only feasible coalition that the members agreed on. Finally, we can show that at least one agent will get a strictly better payoff. Either $N_{\mathcal{C}} = N_{\mathcal{C}'}$ (and since $\phi_{\mathcal{C}} \neq \phi_{\mathcal{C}'}$, some agent must get a different payoff) or $N_{\mathcal{C}} \neq N_{\mathcal{C}'}$ (and the agents from $N_{\mathcal{C}'} \setminus N_{\mathcal{C}}$ will get a positive payoff).

$\Longrightarrow$ Assume that $\mathcal{C}$ is not explicitly nor implicitly endangered by any coalition. First, if the agents from $N_{\mathcal{C}}$ make the collaborative action $\mathcal{C}$, then they will all get positive payoffs. Indeed, the agents in $N_{\mathcal{C}}$ could not get positive payoffs only if there would exist a cheaper feasible coalition $\mathcal{C}'$ such that $N_{\mathcal{C}} \cap N_{\mathcal{C}'} = \emptyset$. This would, however mean that $\mathcal{C}$ is explicitly endangered by $\mathcal{C}'$. Next, we show that the state in which the agents from $N_{\mathcal{C}}$ make the collective decision $\mathcal{C}$ and the other agents play arbitrary actions is RSNE. For the sake of contradiction let us assume that there exists a subset of agents $N_{\mathcal{C}'}$ which can make a collaborative action $\mathcal{C}'$ after which the payoff of everyone from $N_{\mathcal{C}'}$ would be at least equal to her payoff in $\mathcal{C}$. This would, however mean that $\mathcal{C}$ is either implicitly or explicitly endangered by $\mathcal{C}'$. This completes the proof. □

The result in Theorem 8 stated for RSNE transfers to SNE with a slight modification of the model that associates some small costs of preparing a bid by agents. To state the result for SNE we also need to use the definition of a coalition $\mathcal{C}$ being *strictly implicitly endangered* by $\mathcal{C}'$ (here we do not require the agents from $N_{\mathcal{C}} \cap N_{\mathcal{C}'}$ to have at least as good payoffs, but strictly better payoffs in $\mathcal{C}'$ than in $\mathcal{C}$).

THEOREM 8. *If there are small but positive costs of preparing the offer by the agents then the coalition $\mathcal{C}$ is strongly winning if and only if $\mathcal{C}$ is not explicitly nor strictly implicitly endangered by any coalition.*

PROOF. Analogous to the proof of Theorem 7. □

Theorems 7 and 8 give us better understanding of the concept of Rigorously Strong Nash Equilibrium (and Strong Nash Equilibrium) in our model. They also lead to a simple brute-force algorithm for checking whether the coalition can be a part of some RSNE. Below, we provide the analysis that allows to characterize RSNE in the project salary model even more precisely.

THEOREM 9. *In the project salary model the set of agents participating in a rigorously strongly winning coalition is the same as the set of agents participating in the cheapest feasible coalition.*

There is no analogous result for the hourly salary model.

PROPOSITION 10. *In the hourly salary model the set of the agents participating in a rigorously strongly winning coalition may not be the same as the set of the agents participating in the cheapest feasible coalition.*

PROPOSITION 11. *In the project salary model the bid of a strongly winning coalition is equal to the maximal allowed price v.*

Theorem 9 and Proposition 11 show that the problem of finding a strongly winning coalition collapses to the problem of finding a feasible coalition. The problem thus becomes an optimization problem; the strategic behavior of agents does not have an influence on this procedure.

PROPOSITION 12. *Checking whether a coalition is rigorously strongly winning can be solved in time $O(n^2 \cdot \mathit{fcfc})$, where fcfc is the complexity of the problem* FCFC.

PROPOSITION 13. *In the project salary model, if the salaries of the agents can be rational numbers, finding a rigorously strongly winning coalition can be solved in time $O(n^5 \log(nv)\mathit{fcfc})$, where fcfc is the complexity of* FCFC.

It is desired to know Rigorously Strong Nash Equilibrium provided they exist. However a RSNE (and even a Strong Nash Equilibrium) may not exist in some instances.

PROPOSITION 14. *Both in the project salary and in the hourly salary model, there may not exist a strongly winning coalition even though there exists a feasible coalition.*

PROOF. Consider a project with budget $v = 5$; and three identical agents a, b, c with minimal salaries $\phi_i^{\min} = 2$ (in the hourly salary model, assume that each agent spends exactly 1 time unit on the project). The deadline is $d = 1$; a coalition of any two agents is feasible (able to complete the project before the deadline and within the budget).

For the sake of contradiction assume there exists a coalition $\mathcal{C}$ that gets positive payoffs. Without loss of generality we assume that $N_{\mathcal{C}} = \{a, b\}$. At least one of the agents, let us say a has to get salary equal to 2. However, the agents a and c, with the salaries equal to 3 and 2 respectively, can form a feasible coalition in which both a and c get better payoffs (note that we use here the fact that the payoffs are discrete). □

4.2 Weakly winning coalitions

We are not fully satisfied with the example from Proposition 14. Indeed the coalition $\{a, c\}$ can profit by deviating, but a should not be wiling to deviate. The reason is that $\{a, c\}$ is not stable by its own, and can be successfully deviated by $\{b, c\}$. Thus, we propose a weaker notion.

DEFINITION 5. *A feasible coalition $\mathcal{C}$ is* weakly winning *if it is not explicitly endangered by any coalition and for each feasible coalition $\mathcal{C}'$ such that $\mathcal{C}$ is implicitly endangered by $\mathcal{C}'$, there exists a feasible coalition $\mathcal{C}''$ such that $\mathcal{C}'$ is explicitly or implicitly endangered by $\mathcal{C}''$.*

Another way of weakening the notion of the (rigorously) strongly winning coalition is to consider Coalition-Proof Equilibria [11]. We also believe that it is an interesting open question to find whether there is some relation between the concept of the weakly winning coalition and CPE.

PROPOSITION 15. *There exists a weakly winning coalition if and only if there exists a feasible coalition.*

PROOF. Consider a feasible coalition $\mathcal{C}$ that is not explicitly endangered (such a coalition exists provided there exists a feasible coalition). Let $\mathcal{E}$ denote a set of feasible coalitions implicitly endangering $\mathcal{C}$. If $\mathcal{E} = \emptyset$, $\mathcal{C}$ is strongly winning and, thus also, weakly winning. If there exists $\mathcal{C}' \in \mathcal{E}$ such that $\mathcal{C}'$ is not (implicitly or explicitly) endangered by any feasible coalition, then $\mathcal{C}'$ is strongly winning (and, thus also, weakly winning). Otherwise, $\mathcal{C}$ is weakly winning.

If there is no feasible coalition then there is no weakly winning coalition. □

PROPOSITION 16. *In the project salary model, if the salaries of the agents can be rational numbers, the problem of finding a weakly winning coalition can be solved in time $O(n^5 \log(nv)\mathit{fcfc})$, where fcfc is the complexity of* FCFC.

PROPOSITION 17. *In the project salary model, if the salaries of the agent can be rational numbers, the problem of checking whether a coalition $\mathcal{C}'$ is weakly winning coalition can be solved in time $O(n^5 \log(nv)\mathit{fcfc})$, where fcfc is the complexity of* FCFC.

5. MECHANISM DESIGN

In this section we take a look at two mechanisms that a project issuer can apply to find a winning team: the first one sets the job's budget v; the second one uses an English auction.

First, we show that if the client is allowed to change the value v there exists a simple mechanism ensuring the existence of the strongly winning coalition.

THEOREM 18. *If there exists a feasible coalition, then there exists a budget v^* for which there exists a strongly winning coalition. The problem of finding such v^* can be solved in time $O(\log v \cdot \mathit{ffc})$, where ffc is the complexity of* FFC.

PROOF. Let v^* be the smallest value such that there exists a feasible coalition. We show that for v^* there exists a strongly winning coalition. Let $\mathcal{C}^*$ be the most preferred (according to the tie-breaking rule $\prec$) feasible coalition for v^*. For the sake of contradiction let us assume that there exists a coalition $\mathcal{C}'$ such that $\mathcal{C}^*$ is strictly implicitly or explicitly endangered by $\mathcal{C}'$. Of course $b_{\mathcal{C}'} \leq v^*$ (otherwise $\mathcal{C}'$ would not be feasible). If $\mathcal{C}^*$ is explicitly endangered by $\mathcal{C}'$ ($N_{\mathcal{C}^*} \cap N_{\mathcal{C}'} = \emptyset$), it means $\mathcal{C}'$ is cheaper than $\mathcal{C}^*$; and we get a contradiction with the definition of v^*. Otherwise ($\mathcal{C}^*$ is strictly implicitly endangered by $\mathcal{C}'$), let $i \in N_{\mathcal{C}^*} \cap N_{\mathcal{C}'}$. Now, i must get strictly better salary in $\mathcal{C}'$ than in $\mathcal{C}^*$. Thus if we change the salary of i in the coalition $\mathcal{C}'$ to $\phi_{\mathcal{C}'}(i) = \phi_{\mathcal{C}^*}(i)$ we get a contradiction—a cheaper feasible coalition. □

In the second approach we use the English auction in which coalitions participate. As in a standard English auction, the auction starts from the least preferred outcome for the client (the original budget v); the asking price is gradually *decreased*. Coalitions place bids for the current asking price. The auction stops if there is no feasible coalition that can propose a lower bid than the current asking price. This leads to the concept of an auction-winning coalition.

DEFINITION 6. *A coalition $\mathcal{C}$ is* auction-winning *if and only if there is no feasible coalition $\mathcal{C}'$ such that $b_{\mathcal{C}'} < b_{\mathcal{C}}$ and for each agent $i \in N_{\mathcal{C}} \cap N_{\mathcal{C}'}$, i gets better salary in $\mathcal{C}'$, $\phi_{\mathcal{C}'}(i) \geq \phi_{\mathcal{C}}(i)$.*

PROPOSITION 19. *The problem of checking whether a feasible coalition $\mathcal{C}$ is auction-winning can be solved in time $O(\mathit{ffc})$. The problem of finding an auction-winning coalition can be solved in time $O(v \cdot \mathit{ffc})$; ffc is the complexity of* FFC.

The summary of our results in general model is given in Table 1. We believe that the computational results favor the concept of the auction winning coalition (or the centralized model). First, the weakly winning coalition is guaranteed to exist. Second, the computational power needed to find an auction winning coalition seems much smaller in comparison with other concepts. In the centralized model, finding the winning coalition (when we already have the asking salaries of the agents) has also a straightforward reduction to FCFC.

		Exist	Checking	Finding
Decentr.	RSW	no	$O(n^2 \cdot \mathit{fcfc})$	$O(n^5 \log(nv)\mathit{fcfc})$ (*-)
	SW	no	open problem	
	WW	yes	$O(n^5 \log(nv)\mathit{fcfc})$ (*-)	
	AW	yes	$O(\mathit{ffc})$	$O(v \cdot \mathit{ffc})$
Centr.	WC	N/A	$O(\mathit{fcfc})$	
	SNE	yes (*) no (+)	$O(\mathit{fcfc})$	$O(n^3 \log(nv)\mathit{fcfc}))$ (*-)

Table 1: The summary of the results in general model. The column "Exist" contains the information whether a coalition/equilibrium always exists. The column "Checking" contains the complexity of checking whether a given coalition satisfies the definition corresponding to the row. The column "Finding" contains the complexity of finding a coalition/equilibrium (*ffc* and *fcfc* are the complexities of the problems FFC and FCFC, respectively). The values marked as (*) are valid only in the project salary model; marked as (+) only in the hourly salary model; marked as (-) only if the salaries of the agents can be rational numbers. In the table: (R)SW = (rigorously) strongly winning, WW = weakly winning, AW = auction winning, WC = winning (provided we have asking salaries) coalition, SNE = Strong Nash Equilibrium.

6. FFC IN A SCHEDULING MODEL

In Sections 3, 4 and 5 we show that many problems of finding the (weakly/strongly) winning coalitions or determining whether a given coalition is (weakly/strongly) winning required solving the subproblem of finding the feasible coalition. The general model (Section 2) assumed that given a coalition there is an oracle deciding whether the coalition can finish the project before the given deadline. In this section, we show a possible concrete instance of this model in which a project is a set of indivisible, independent tasks; and agents are processors who process these tasks with varying speeds.

6.1 The scheduling model

A project consists of a set $\mathcal{T} = \{t_1, t_2, \ldots, t_q\}$ of q independent tasks. The tasks can be processed sequentially or in parallel. The tasks are indivisible: a task must be processed on a single processor. Once started, a task cannot be interrupted. All tasks must be completed before d, the project's deadline.

Agents correspond to machines (processors) processing tasks $\mathcal{T}$ (in this section we use the words the agent and the machine interchangeably). Each agent has certain skills which are represented as the speed of executing the tasks. Thus, for each agent i we define the skill vector $s_i = \langle s_{i,1}, s_{i,2}, \ldots s_{i,q} \rangle$ which has the following meaning: agent i is able to finish task t_j within $s_{i,j}$ time units (with $s_{i,j} = \infty$ when an agent is unable to finish the task). We assume that s_i is known (it can be well approximated from e.g. past behavior of the agent certified by clients in form of reviews). An agent can process only a single task at each time moment—if she wants to process more than one task, she must execute the tasks sequentially. We assume that only a single agent can work on a given task. This assumption is not as restrictive as it may appear; if the task t_i is large and can be processed by multiple agents in parallel, the project client will rather replace t_i by a number of smaller tasks.

For a coalition $\mathcal{C}$ we define $\Phi_{\mathcal{C}} : \mathcal{T} \to N_{\mathcal{C}}$ to be an assignment function (assigning tasks to agents). The assignment function $\Phi_{\mathcal{C}}$ enables us to formalize the notion of a coalition completing the project before the deadline and also the total cost of the coalition. Specificaly, a project is finished before the deadline d if and only if all the agents finish their assigned tasks before d, $\forall i \in N_{\mathcal{C}} \sum_{\ell:\Phi(t_\ell)=i} s_{i,\ell} \leq d$. In the hourly salary model, the cost of the coalition is equal to $c_{\mathcal{C}} = \sum_{i \in N_{\mathcal{C}}} \phi_{\mathcal{C}}(i) \sum_{\ell:\Phi(t_\ell)=i} s_{i,\ell}$.

In the scheduling model the problem of finding a feasible coalition can be defined as follows.

PROBLEM 3. (FFCSM: FIND FEASIBLE COALITIONS, SCHEDULING MODEL). *Let $\mathcal{T}$ be the set of q tasks and N be the set of machines (or equivalently, agents). For each task $t_j \in \mathcal{T}$ and each machine (agent) $i \in N$ we define $s_{i,j}$ as the processing time of t_j on i. Let $\phi_i^{\min}$ be the cost of renting machine i (hiring agent i). The budget of the project is v and the deadline is d. The FFCSM problem consists of selecting a subset of the machines $N' \subseteq N$ and the assignment function $\Phi : \mathcal{T} \to N'$ such that the budget is not exceeded ($c_{N',\Phi} \leq B$) and the project's makespan does not exceed the deadline d.*

In the hourly salary model, the problem of finding the feasible coalition reduces to the problem of scheduling on unrelated machines with costs [12]. Specifically, Shmoys and Tardos [12] show a 2-approximation algorithm for approximating the makespan (the deadline d in our model).

PROBLEM 4. (FFCHS: FIND FEASIBLE COALITIONS, HOURLY SALARY). *The instance of the problem is the same as in the* FFCSM *problem. In the* FFCHS *problem we additionally specify that the cost of the coalition $c_{N',\Phi}$ is defined as $c_{\mathcal{C}} = \sum_{i \in N_{\mathcal{C}}} \phi_{\mathcal{C}}(i) \sum_{\ell : \Phi(t_\ell) = i} s_{i,\ell}$.*

The project salary model is a generalization of the problem of minimizing makespan on unrelated machines [13]. To the best of our knowledge this problem has not been stated before; thus we formally define it below.

PROBLEM 5. (FFCPS: FIND FEASIBLE COALITIONS, PROJECT SALARY). *The instance of the problem is the same as in the* FFCSM *problem. In the* FFCPS *problem we additionally specify that $c_{N',\Phi}$ is defined as $c_{N',\Phi} = \sum_{i \in N'} \phi_i^{\min}$.*

FFCPS is similar to the problem of fiding a fully proportional representation [14, 15]. Fiding a fully proportional representation is connected to resource allocation [16]; in fact, its utilitarian version can be viewed as a special case of FFCPS, where the goal is to minimize the flow time of the tasks instead of a makespan. An easier problem in which the goal is to optimize the assignment only (assuming that the machines are already selected) has a 2-approximation algorithm [13]. However, adding the notion of the budget usually significantly increases the complexity. E.g., in case of fully proportional representation, the problem of the finding optimal assignment is polynomial [17], but the full problem is computationally hard. However, there are good approximation algorithms known [18, 16, 19]. Thus, we ask for the approximability of FFCPS too.

6.2 FFCPS: Hardness Results

First, we show the NP-hardness of FFCSM in restricted special cases.

THEOREM 20. FFCPS *and* FFCHS *are NP-hard even for two agents.*

THEOREM 21. FFCPS *is NP-hard even if the agents can be assigned no more than 3 tasks, if each agent has no more than 3 skills (for each j we have that $\|\{i : s_{i,j} \neq \infty\}\| \leq 3$), if the deadline is constant, and if the minimal salaries of the agents are equal to 1.*

THEOREM 22. FFCHS *is NP-hard even if the agents can be assigned no more than 4 tasks, if each agent has no more than 4 skills (for each j we have that $\|\{i : s_{i,j} \neq \infty\}\| \leq 4$), if the deadline is constant, and if the minimal salaries of the agents are equal to 1.*

PROOF. The proof is by reduction from the exact set cover problem. We are given a set of elements $T = \{t_1, t_2, \dots, t_q\}$ and family $\mathcal{S} = \{S_1, S_2, \dots, S_n\}$ of 3-element subsets of T. We assume that each member of T appears in at most 3 sets from $\mathcal{S}$.

We build an instance I of the feasible coalition problem in the following way. There are $q + n$ tasks and $2n$ agents. The first q tasks $t_1, t_2, \dots, t_q$ correspond to the elements in T. The next n tasks $t_{q+1}, t_{q+2}, \dots t_{q+n}$ are the dummy tasks needed by our construction. The first n agents $1, 2, \dots, n$ correspond to the subsets from $\mathcal{S}$ and the next n agents $(n+1), (n+2), \dots, 2n$ are the dummy agents. The minimal salaries of all agents are equal to 1.

For each agent i, $i \leq n$ and each task t_j, $j \leq q$, we set $s_{i,j} = 2$ if and only if $t_j \in S_i$; otherwise $s_{i,j} = \infty$. Also, for each agent i, $i \leq n$ and each task t_j, $j > q$ we set $s_{i,j} = 5$ if and only if $i = j - q$; otherwise $s_{i,j} = \infty$. For each agent i, $i > n$ and each task t_j we set $s_{i,j} = 6$ if and only if $i - n = j - q$; otherwise $s_{i,j} = \infty$. The deadline d is equal to 6 and the budget v is equal to $v = \frac{7}{3}q + 5n$. Clearly, each agent has no more than 4 skills and so, in any feasible solution, cannot be assigned more than 4 tasks.

We will show that the answer to the original instance of the exact set cover problem is "yes" if and only if there exists a feasible coalition in the our constructed instance I.

$\Longleftarrow$ Let us assume there exists a feasible coalition $\mathcal{C}$. The cost of this coalition is at most equal to $v = \frac{7}{3}q + 5n$. Each non-dummy task (there are q such tasks) takes 2 time units, and thus implies the cost equal to 2. The dummy tasks can be assigned either to non-dummy agents (implying the cost 5) or to dummy agents (implying the cost 6). Thus, we infer that at most $\frac{q}{3}$ dummy agents are assigned a task ($2q + \frac{1}{3}q \cdot 6 + (n - \frac{1}{3}q) \cdot 5 = v$). As the result at least $(n - \frac{q}{3})$ dummy tasks must be assigned to non-dummy agents. A non-dummy agent, who is assigned a dummy task cannot be assigned any other task (otherwise the completion time would exceed the deadline). Thus, at most $\frac{q}{3}$ non-dummy agents can be assigned non-dummy tasks. The non-dummy tasks can be assigned only to non-dummy agents. We see the subsets corresponding to these non-dummy agents who are assigned non-dummy tasks form the solution to the initial exact set cover problem.

$\Longrightarrow$ Let us assume that there exists the exact set cover in the initial problem. The agents correponding to the subsets from the cover can be assigned tasks so that the deadline is not exceeded and the total cost of completing these tasks is equal to $2q$. The other $(n - \frac{q}{3})$ non-dummy agents can be assigned one dummy task each. Finally, not-yet assigned dummy tasks can be assigned to dummy agents. The total cost of such assignment is equal to $2q + (n - \frac{1}{3}q) \cdot 5 + \frac{1}{3}q \cdot 6 = v$.

This completes the proof. □

Unfortunately, FFCPS is not approximable for makespan, for budget, and even for the combination of both parameters.

THEOREM 23. *For any $\alpha, \beta \geq 1$ there is no polynomial α-β-approximation algorithm for* FFCPS *that approximates makespan with the ratio α and budget with the ratio β, unless P=NP. This result holds even if the costs of all machines are equal 1.*

Theorems 20, 21, and 22 show that the problems FFCPS and FFCHS remain NP-hard even if various parameters are constant. Although Theorem 20 give us NP-hardness even for 2 agents, it is somehow not satisfactory as we used the fact that the deadline d can be very large. If the deadline is given in unary encoding, we can solve the case for 2 agents by dynamic programming. Thus, we asked the question, whether we can solve the problem efficiently for small number of agents, if the size of the input is given in unary encoding. We used the parametrized complexity theory [20] to approach this problem: we asked whether FFCPS and FFCHS belong to FPT for the parameter n—the number of the agents, provided the input is given in unary encoding. Unfortunately, FFCPS and FFCHS are also hard for the parameter n.

THEOREM 24. *Consider the number of the agents as the parameter.* FFCPS *and* FFCHS *are W[1]-hard, even if the agents are the same, if for minimal salaries of the agents equal to 1, and if the size of the input is given in unary encoding.*

PROOF. The proof is by reduction from Unary Bin Packing (which is W[1]-hard [21]). $\square$

6.3 Integer programming formulation

In the hourly salary model, Shmoys and Tardos [12] show an integer programming formulation. In this subsection we state the FFCPS problem as an integer program.

$$\text{minimize } d \tag{1}$$

$$\sum_{i \in N} a_i \phi_i^{\min} \le v \tag{2}$$

$$\forall_{i \in N; t_j \in T} \quad x_{i,j} \le a_i \tag{3}$$

$$\forall_{i \in N} \quad \sum_{t_j \in T} x_{i,j} s_{i,j} \le d \tag{4}$$

$$\forall_{i \in N; t_j \in T} \quad x_{i,j} \in \{0, 1\} \tag{5}$$

$$\forall_{i \in N} \quad a_i \in \{0, 1\} \tag{6}$$

In the above formulation, a binary variable a_i denotes whether agent i is a part of the solution (is assigned some tasks, Equation 6). A binary variable $x_{i,j}$ is equal to 1 if and only if the task t_j is assigned to the agent i (Equation 5). We minimize the makespan d (Equation 1), which is the maximal completion time of the tasks over all the agents (Inequation 4). We cannot exceed the budget v (Inequation 2), and the tasks can be assigned only to the selected agents (Inequation 3).

7. CONCLUSIONS

In this paper we present a new class of the coalition games that model cooperation and competition between agents for the employment in a complex project. We believe that this is an interesting setting that relates to other natural problems, like coalition formation, coalitional auctions, auctions for sharable items, etc. We consider two models of the organization of the market. First, the winning coalition is selected by a central mechanism; the agents are strategic about the salaries they ask. Second, the coalition formation process is decentralized—the already-formed coalitions bid for the project, thus the agents are strategic both about the salaries and their cooperation partners.

We propose the concepts of stability in each model. These concepts are of interest both to the agents and to the client. The client gains an insight into agents' strategies and can thus establish a relation between the cost of organizing the market and the cost of the winning coalition. The agents can optimize their strategies according to their beliefs (an agent can ask e.g. whether she can increase her asking salary and still participate in the winning coalition). In the centralized model we show that the Strong Nash Equilibrium always exist. In the decentralized model the SNE may not exist, but we prove the existence of a weakly winning coalition. We show how to reduce the problem of finding a winning coalition to the problem of finding a feasible one. Finally, we show a concrete model in which the project is represented as a set of independent tasks and the agents have certain skills (expressed as the processing speeds). We prove the hardness of the problems in restricted cases.

All the omitted proofs are available in the full version of the paper, which we provide in an anonymized manner at: `www.dropbox.com/s/74la1mz03x1m9x9/crowdsourcing.pdf`

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