Foundations of mathematics – week 13
January 15, 2010

Exercises

1. Let $B \subseteq A \times A$. Prove that there exists a maximal (with respect to inclusion) subset $C \subseteq A$ such that $C \times C \subseteq B$.

2. Let $B \subseteq \mathbb{R}_+$. Prove that there exists a set $C \subseteq \mathbb{R}$ such that $\forall x, y \in C(x \neq y \rightarrow |x - y| \in B)$ and $\forall x(x \notin C \rightarrow \exists y \in C|x - y| \notin B)$.

3. Prove that every partial order can be extended to a total order.

Homework

1. Any subset of the set $\mathbb{Z}$ is called a testimony. The set of testimonies R is inconsistent if and only if there exists $i \in \mathbb{Z}$ such that $i, -i \in \bigcup R$. Prove that if R is any family of testimonies then there exists a maximal inconsistent family of testimonies $R' \subseteq R$.

2. Let $D \subseteq A \times A$. Prove that there exists a set $X \subseteq A$ such that $Z \times Z \cap D = \emptyset$ and if $Z \subsetneq V \subseteq A$ then $(V \times V) \cap D \neq \emptyset$.