Foundations of mathematics – week 12
January 8, 2010

Exercises

1. Find the cardinality of the Cantor set.
2. Equivalence relation R in the set $\mathbb{N}^{\mathbb{N}}$ is defined in the following way
$$R = \{\langle f, g\rangle \mid \forall n f(2n) = g(2n)\}.$$
Find the cardinality of the set of all equivalence classes of the relation R and the cardinality of each equivalence class.
3. Find the minimal, maximal, least and greatest elements in the set $\{2, 3, 4, 5, 6, 8, 9, 12, 24\}$ ordered by divisibility. Are there any three-element chains or antichains in the set?
4. Find the minimal, maximal, least and greatest elements in the set
$$\{\{1, 2, 3, 4, 6\}, \{3\}, \{1, 2, 3, 4, 5\}, \{2, 3, 5\}, \{1, 2, 3, 4\}, \{1, 2\}\}$$
ordered by inclusion.
5. Give an example of a partially ordered set which has two maximal elements, one minimal element and no least element.
6. Give an example of a partially ordered set which has two maximal elements, one minimal element, no least element and a four-element antichain which is bounded from above but does not have an upper bound .
7. Does the set $\{01^n \mid n \in \mathbb{N}\}$ have an upper (lower) bound in the set $\{0, 1\}^*$ ordered lexicographically?
8. Does the set $\{0^n 1 \mid n \in \mathbb{N}\}$ have an upper (lower) bound in the set $\{0, 1\}^*$ ordered lexicographically?
9. How many equivalence relations in $\mathbb{N}$ which are also partially ordered sets are there?

Homework

1. Let $\leqslant$ be a partial order in A. The relation $<$ is the called a strict order induced by $\leqslant$. Show the strict orders induced by partial orders are exactly the relations which are transitive and irreflexive.
2. Does the set of all words ovet the alphabet $\{0, 1\}$ which have an equal number of zeros and ones have an upper (lower) bound in the set $\{0, 1\}^*$ ordered lexicographically?