

Foundations of mathematics – week 10

December 11, 2009

Exercises

1. Show that $\mathbb{N} \times \mathbb{N} \sim \mathbb{N}$.
2. Show that $\mathbb{Q} \sim \mathbb{N}$.
3. Show that if $A \sim B$ then $P(A) \sim P(B)$.
4. Show that $\mathbb{R} \not\sim \mathbb{N}$.
5. What is the cardinality of the set of all intervals with rational endpoints?
6. Find the cardinality of the set $P_{fin}(\mathbb{N})$ of all finite subsets of \mathbb{N} .
7. Prove that if A is an arbitrary set of disjoint open intervals in \mathbb{R} then $\overline{A} \leq \aleph_0$.
8. Prove that the set of points of discontinuity of a monotonic function from \mathbb{R} to \mathbb{R} is countable.

Homework

1. Find the cardinality of the set of all sequences of rational numbers which are constant from a certain point.
2. Let equivalence relation $r \subseteq \mathbb{R}^2$ be such that

$$\forall x \in \mathbb{R} \exists \epsilon > 0 ((x - \epsilon, x + \epsilon) \subseteq [x]_r).$$

What can you say about the cardinality of the set $\mathbb{R}/_r$?