Foundations of mathematics – week 9

December 4, 2009

Exercises

1. Is it always true that
$$\bigcup_{i\in I}(A_i \cap B_i) = \bigcup_{i\in I}\bigcup_{j\in I}(A_i \cap B_j)?$$
2. Let $A_{n,m} = \{x \in \mathbb{R} \mid \frac{n-1}{m+1} \leqslant x < n+m\}$. Find $\bigcup_{n\in\mathbb{N}}\bigcap_{m\in\mathbb{N}} A_{n,m}$ and $\bigcap_{n\in\mathbb{N}}\bigcup_{m\in\mathbb{N}} A_{n,m}$.
3. Which of the following equlities hold for arbitrary set $A_{t,s}$ where $t \in T$, $s \in S$
   (a) $\bigcup_{t\in T}\Pi_{s\in S} A_{t,s} = \Pi_{s\in S}\bigcup_{t\in T} A_{t,s}$?
4. The function $F : P(\mathbb{N})^{\mathbb{N}} \to P(\mathbb{N})$ is such that
$$F(x) = \bigcup\{x(i) \mid i \in \mathbb{N}\}.$$
   (a) Is F injective?
   (b) Is F onto $P(\mathbb{N})$?
   (c) Does there exist a set $A \subseteq \mathbb{N}$ such that $F^{-1}(\{A\})$ is a singleton?
   (d) Does there exist a set $A \subseteq \mathbb{N}$ such that $F^{-1}(\{A\})$ is a four element set?