Foundations of mathematics – week 6

November 13, 2009

Exercises

1. Is it true that for an arbitrary A and for an arbitrary relation R

$$I_A \subseteq R^{-1}; R?$$

2. Give an example of a 5-element relation on the set of natural numbers such that R is
   (a) symmetric;
   (b) reflexive;
   (c) transitive.
3. Is the relation $\{\langle 0,3\rangle, \langle 1,3\rangle, \langle 1,5\rangle, \langle 4,5\rangle, \langle 4,2\rangle\}$ transitive?
4. Is it possible:
   (a) $R^{-1} \subsetneq R$;
   (b) $R^{-1} = A^2 - R$?
5. Prove that the relation R is transitive if and only if $R; R \subseteq R$.
6. How many total functions, partial functions, injective functions and surjective functions are there in:
   (a) $\emptyset \to \emptyset$;
   (b) $\{\cdot\} \to \emptyset$;
   (c) $\emptyset \to \{\cdot\}$;
   (d) $\{\cdot\} \to \{\cdot\}$;
   (e) $\{\cdot, \square\} \to \{\cdot\}$;
   (f) $\{\cdot\} \to \{\cdot, \square\}$?
7. Prove that if $f : A \xrightarrow{1-1} B$ and $f : B \xrightarrow{1-1} C$ then $g \circ f : A \to C$ is injective.
8. Let $f : A \to B$. Prove that f is injective if and only if for any C for any pair of functions $g, h : C \to A$ the following condition holds

$$f \circ g = f \circ h \to g = h.$$

Homework

1. Let $\mathcal{R}$ be a nonempty family of transitive relations in the set A such that for any $r, s \in \mathcal{R}$, $r \subseteq s$ or $s \subseteq r$. Prove that $\bigcup \mathcal{R}$ is a transitive relation.
2. Let $f : A \to B$. Prove that f is onto B if and only if for any set C for any $g, h : B \to C$ the following condition holds

$$g \circ f = h \circ f \to g = h.$$