Foundations of mathematics – week 5
November 6, 2009

Exercises

1. Which of the implications holds for arbitrary sets A, B?
$$A \subseteq B \leftrightarrow P(A) \subseteq P(B)$$
2. Is it true that if $A \subseteq B$ then $\bigcup A \subseteq \bigcup B$?
3. Is it true that if $\bigcup A \subseteq \bigcup B$ then $A \subseteq B$?
4. Show that for arbitrary set families $\mathcal{A}, \mathcal{B}$ we have $\bigcup(\mathcal{A} \cup \mathcal{B}) = \bigcup \mathcal{A} \cup \mathcal{B}$.
5. Is it true that for arbitrary set $\mathcal{A}, \mathcal{B}$ the following equality holds $\bigcap \mathcal{A} \cup \bigcap \mathcal{B} = \bigcap(\mathcal{A} \cup \mathcal{B})$?
6. Show that $\bigcup P(A) = A$ for an arbitrary A.
7. When $A \times B = B \times A$?
8. Is it true that $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$?
9. Is it true that for each A and for each relation $R \subseteq A \times A$
$$R^{-1}; R \subseteq I_A?$$

Homework

1. Let $A \subseteq P(\mathbb{R})$ be a set family such that
$$\forall B \in A \forall C \subseteq \mathbb{R}(C \subseteq B \rightarrow C \in A).$$
Show that $\bigcup A = \{z \in \mathbb{R} \mid \{z\} \in A\}$.
2. Which of the following equlities
   (a) $\bigcap A \cap \bigcap B = \bigcap(A \cup B)$;
   (b) $\bigcap A \cap \bigcap B = \bigcap(A \cap B)$;
   (c) $\bigcup A \cap \bigcup B = \bigcup(A \cap B)$?

   hold for arbitrary nonempty set families A, B such that $A \cap B \neq \emptyset$?