

## Foundations of mathematics – week 3

October 23, 2009

### Exercises

1. Prove the following formulas.

(a)  $(p \wedge q \rightarrow r) \rightarrow (p \rightarrow q \rightarrow r)$

(b)  $(p \rightarrow q \vee r) \rightarrow (p \rightarrow q) \vee r$

(c)  $\neg \forall x P(x) \leftrightarrow \exists x \neg P(x)$

2. Are the following formulas first order tautologies? Prove or disprove.

(a)  $\exists y \forall z (P(y) \rightarrow Q(z)) \rightarrow (\exists y P(y) \rightarrow \forall z Q(z))$

### Homework

1. Are the following formulas first order tautologies? Prove or disprove.

(a)  $\exists x \exists y (Q(x) \rightarrow P(y)) \rightarrow (\forall x Q(x) \rightarrow \exists y P(y))$

(b)  $\forall x \exists y (P(x) \vee Q(x, y)) \rightarrow \forall x P(x) \vee \forall x \exists y Q(x, y)$

(c)  $\forall x ((P(x) \rightarrow P(y)) \rightarrow Q) \rightarrow Q$