Foundations of mathematics – week 2
October 14, 2009

Exercises

1. Write down the negation of the following statements.
   (a) m and n are prime.
   (b) m are n relatively prime.
2. Why isn't it straightforward to write down the following statements as first order formulas?
   (a) If there exist a rational philosopher then he is a woman.
   (b) The condition $W(x, y)$ holds for every x and for a certain y.
3. What do the following statements mean? Write them down so that it is clear what they mean.
   (a) You cannot drink and play cards.
   (b) The rule applies to people who are Polish citizens and permanently reside in Poland.
   (c) If you don't come or don't call you won't find out.
4. Using the schemas below show that the following formulas are first order tautologies.
   - $p \to q \leftrightarrow \neg p \vee q$
   - $\neg \forall x A(x) \leftrightarrow \exists x \neg A(x)$
   - $\neg \exists x A(x) \leftrightarrow \forall x \neg A(x)$
   - $\forall x(A(x) \wedge B(x)) \leftrightarrow \forall x A(x) \wedge \forall x B(x)$
   - $\exists x(A(x) \vee B(x)) \leftrightarrow \exists x A(x) \vee \exists x B(x)$
   - $\forall x(A \vee B(x)) \leftrightarrow A \wedge \forall x B(x)$, $x \notin FV(A)$
   - $\exists x(A \wedge B(x)) \leftrightarrow A \wedge \exists x B(x)$, $x \notin FV(A)$
   - $A \leftrightarrow \forall x A$, $x \notin FV(A)$
   - $A \leftrightarrow \exists x A$, $x \notin FV(A)$

   (a) $(\exists y p(y) \to \forall z q(z)) \to \forall y \forall z(p(y) \to q(z))$
   (b) $(\forall x \exists y((p(x) \to q(y)) \to r(y))) \to (\forall x p(x) \to \forall y q(y)) \to \exists y r(y)$
5. Are the following formulas first order tautologies?
   (a) $\forall x \exists y P(x, y) \to \exists x \forall y P(x, y)$
   (b) $\forall x(R(x) \to \exists y S(y)) \to \forall x \exists y(R(x) \to S(y))$

Homework

1. Are the following formulas first order tautologies?
   (a) $\exists x(P(x) \to \forall y Q(y)) \to \exists x \forall y(P(x) \to Q(y))$
   (b) $\exists x(\forall y Q(y) \to P(x)) \to \exists x \forall y(Q(y) \to P(x))$
   (c) $\forall x \exists y(P(x) \to R(x, y)) \to \forall x(P(x) \to \exists y R(x, y))$