

## Foundations of mathematics – week 1

October 9, 2009

### Exercises

1. Transform the formula  $(p \rightarrow q) \rightarrow q \rightarrow r$  to conjunctive normal form.
2. How many nonequivalent propositional formulas with one propositional variable are there? with two propositional variables? with  $n$  propositional variables?
3. Write down the following statements using logical symbols and quantifiers and the language of arithmetics  $(\mathbb{N}, +, \cdot, 0, 1, =)$ .
  - (a)  $a$  is less than or equal to  $b$ .
  - (b)  $b$  divided by  $c$  has a remainder  $a$ .
  - (c)  $a$  is prime.
  - (d)  $a$  is the greatest common divisor of  $b$  and  $c$  unless it is prime.
  - (e) No even number is greater than every prime number.
4. Write down the following statements in the language of logic.
  - (a) If some cats are tigers and no tiger is a badger then all badgers have moustache.
  - (b) If every rational philosopher is a cynic and only women are rational then if there exist rational philosophers, some women must be cynic.
5. For each pair of sets write down using the given language the predicate logic formula which is true in one set and false in the other.
  - (a)  $\mathbb{Q}$  and  $\mathbb{R}$ ,  $(+, \cdot, 0, 1, =)$ ;
  - (b)  $\mathbb{N}$  and  $\mathbb{Z}$ ,  $(\leq, 0, =)$ ;
  - (c)  $\mathbb{N}$  and  $\mathbb{Z}$ ,  $(\leq, =)$ .

### Homework

1. Write down the following statements using logical symbols and quantifiers and the language of arithmetics  $(\mathbb{N}, +, \cdot, 0, 1, =)$ .
  - (a) Numbers  $x$  and  $y$  have the same prime divisors..
  - (b) A necessary condition for  $n$  to be even is that  $n$  be divided by 6.
2. For each pair of sets write down using the given language the predicate logic formula which is true in one set and false in the other.
  - (a)  $\mathbb{N}$  and  $\mathbb{Z}$ ,  $(+, =)$ ;
  - (b)  $\mathbb{N}$  and  $\mathbb{Q}$ ,  $(\cdot, =)$ ;