Exam for 'Algorithmics' course, 11.06.2014

Part II: problem solving

1. *In your solutions you can use only theorems, claims, algorithms stated in the lecture or exercises.*
2. *Please deliver the solution of every problem on a separate piece of paper.*
3. *Please justify (prove) your claims.*
4. *You can write in Polish on English but we strongly suggest that Polish students write in Polish.*
5. *Every problem has a simpler version worth 5 points. If you solve the 10 points version you do not need to solve the 5 points version.*

Problem 1 [5 or 10 pts] Describe an algorithm for the following problem. We are given an undirected graph $G = (V, E)$ and a function $f : V \to \{1, \dots, k\}$. Find a minimum-sized subset of vertices $S \subseteq V$ such that after deleting S from G there is no path $v_1, v_2, \dots, v_k$ such that $f(v_i) = i$.

Simpler version: Assume $k = 2$.

Note: Analyze the running time of your algorithm. Your score depends on the running time.

Problem 2 [5 or 10 pts] Design a polynomial-time randomized Monte-Carlo algorithm for the following problem. We are given an undirected n-vertex graph where every vertex is either black or white. We are also given two numbers $k_1, k_2 = O(\log n)$. Find a connected subgraph of G with *exactly* k_1 white vertices and k_2 black vertices, or report that no such subgraph exists.

Simpler version: Find a *path* in G with *exactly* k_1 white vertices and k_2 black vertices, or report that no such subgraph exists.

Note. Analyze the running time of your algorithm and the success probability. The degree of the polynomial in the running time bound does not influence your score.

Problem 3 [5 or 10 pts] Recall that a polynomial-time approximation scheme (PTAS) is an algorithm which takes an instance of an optimization problem and a parameter $\epsilon > 0$ and, in polynomial time, produces a solution that is within a factor $1 + \epsilon$ of being optimal. (The time can depend on ϵ but it should be polynomial for every fixed ϵ.)

Give a PTAS for the following problem. We are given a directed acyclic graph G, two specified nodes s and t of G, an integer L and functions $\ell : E(G) \to \mathbb{N}_+$ and $c : E(G) \to \mathbb{N}_+$. For every edge $(u, v) \in E(G)$, the value $\ell((u, v))$ is the length of (u, v) and $c((u, v))$ is the cost of (u, v). For a path P its length and cost are defined naturally as sum of the lengths and costs of its edges, respectively. The goal is to find a minimum cost path from s to t of length at most L.

Simpler version: Let $c_{\min}$ and $c_{\max}$ denote the minimum and the maximum value of function c. Assume $c_{\max}/c_{\min} \le p(n)$ for some polynomial p, where $n = |V(G)|$.

Note: Analyze the running time of your algorithm and the approximation ratio. The degree of the polynomial in the running time bound does not influence your score.

Good luck!