Optimal Dynamic Strings

Paweł Gawrychowski[2], **Tomasz Kociumaka**[1], Adam Karczmarz[1], Jakub Łącki[3], Piotr Sankowski[1]

[1]University of Warsaw, Poland
[2]University of Wrocław, Poland
[3]Google Research NY, USA

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Dynamic Strings Problem

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- $\texttt{make\_string}(w)$: insert $w \in \Sigma^+$ to $\mathcal{W}$;

$\mathcal{W}$: $\texttt{ab}$

$\texttt{1}$

$$\texttt{make\_string}(\texttt{ab}) = 1$$

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- $\texttt{concat}(w_1, w_2)$: insert $w_1 w_2$ to $\mathcal{W}$ for $w_1, w_2 \in \mathcal{W}$;

$\mathcal{W}$:	ab	abab
	1	2

$$\texttt{concat}(1,1) = 2$$

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$\mathcal{W}$:	ab	abab	a	bab
	1	2	3	4

$$\texttt{split}(2,1) = (3,4)$$

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$$\texttt{equal}(1, 3) = \textbf{false}$$

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$\mathcal{W}$:	ab	abab	a	bab	ba	b
	1	2	3	4	5	6

$$\texttt{split}(4, 2) = (5, 6)$$

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	1	2	3	4	5	6	7

$$\mathtt{concat}(6, 3) = 7$$

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$$\texttt{LCP}(4, 8) = 2$$

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- $\texttt{LCP}(w_1, w_2)$: return the length of the longest common prefix of w_1 and w_2;
- $\texttt{compare}(w_1, w_2)$: lexicographically compare w_1 and w_2.

$\mathcal{W}$:	ab	abab	a	bab	ba	b	ba	baab
	1	2	3	4	5	6	7	8

$$\texttt{compare}(4, 8) = \text{`>'}$$

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Setting

$n =$ total length of strings in the collection
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Unconditional Lower Bound (this work)

No Monte Carlo algorithm (correct with high probability) supports `split`, `concat`, and `equal` operations in $o(\log n)$ amortized time. This is true even if `split` and `concat` invalidate their arguments.

„Folklore” Solution

Karp–Rabin Fingerprints

- $H(w) = H(w') \implies w = w'$ w.h.p.
- $H(u), H(v) \rightsquigarrow H(uv)$ in $\mathcal{O}(1)$ time.

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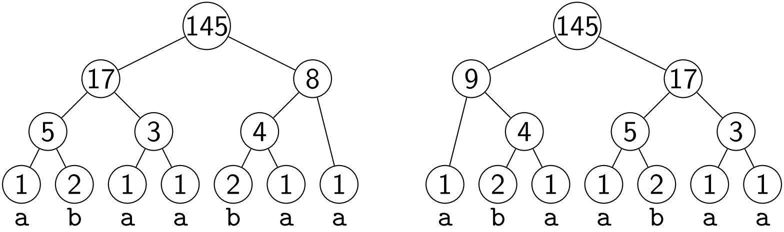

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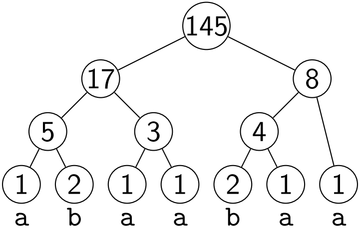

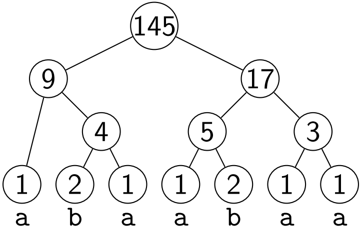

`split`(w, k) $\mathcal{O}(\log |w|)$

`concat`(w_1, w_2) $\mathcal{O}(\log |w_1 w_2|)$

`make_string`(w) $\mathcal{O}(|w|)$

`equal`(w_1, w_2) $\mathcal{O}(1)$

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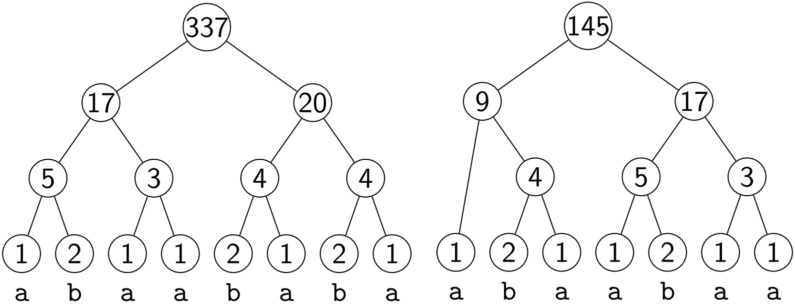

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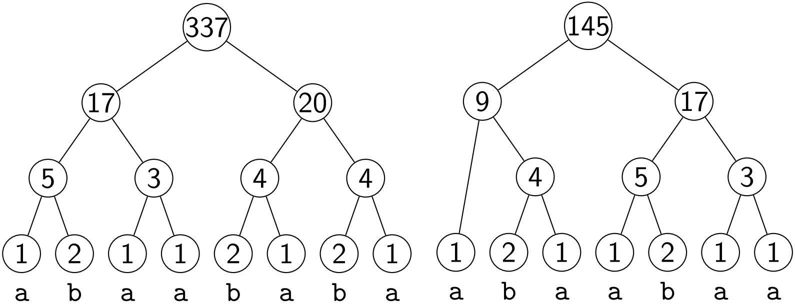

- The BST structure is not particularly helpful...
- Fall back to the naive solution:
  - `LCP`: binary search using `split` and `equal`; $\mathcal{O}(\log^2 |w|)$ time;
  - `compare`: compare the characters following `LCP`.

Consistent Parsing

Consistent Parsing (Mehlhorn et al. SODA'94; Sahinalp–Vishkin STOC'94)

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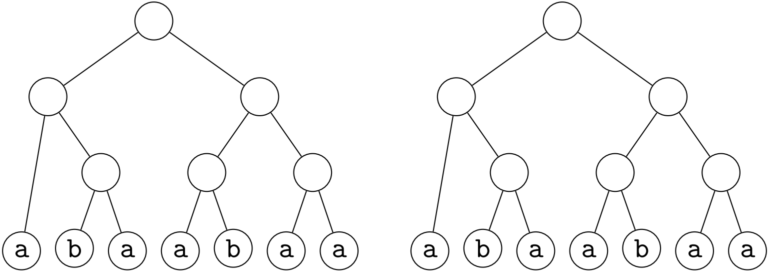

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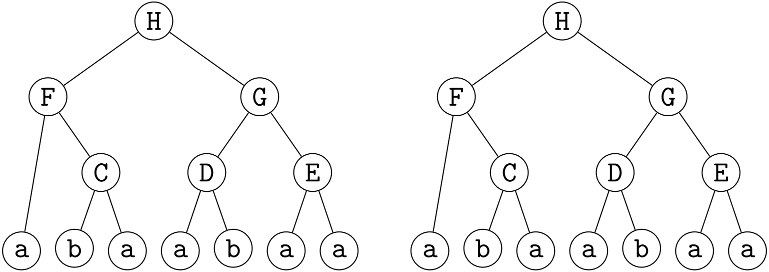

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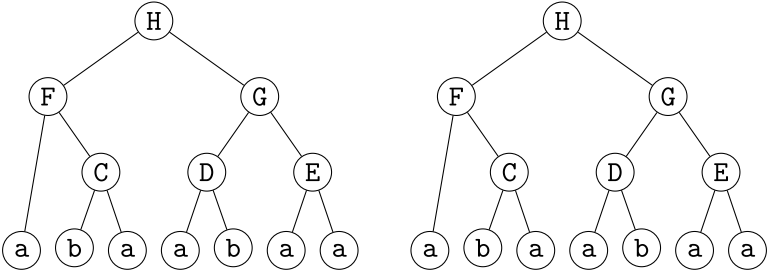

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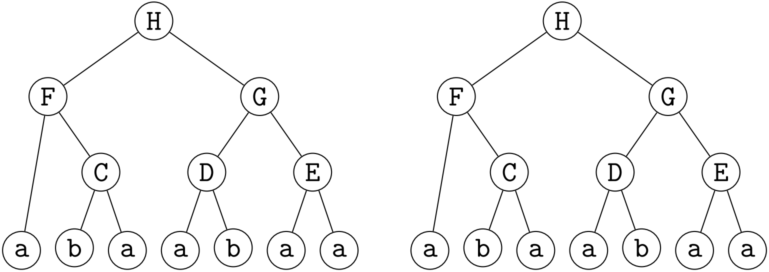

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- Dictionaries needed for the naming function.

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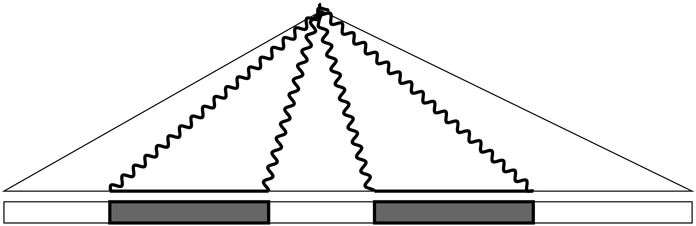

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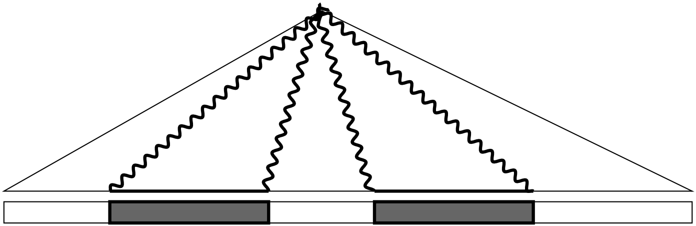

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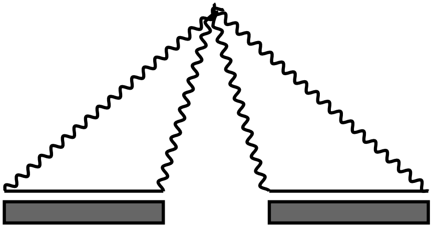

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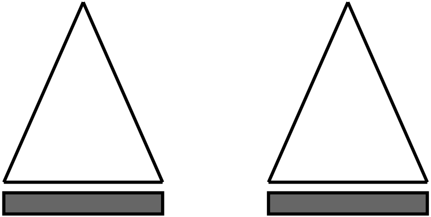

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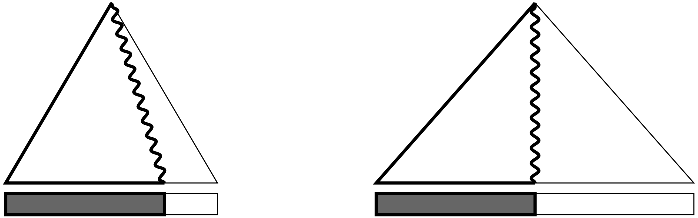

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- allows to maintain consistent parsing,
- enables efficient `LCP` (and `compare`) queries.

Parse Tree Construction: RLE

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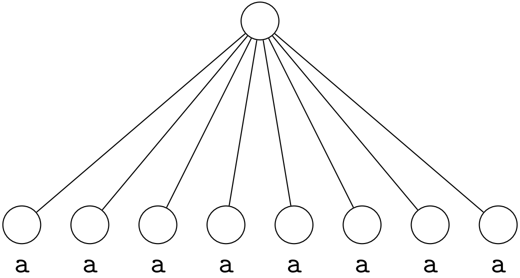
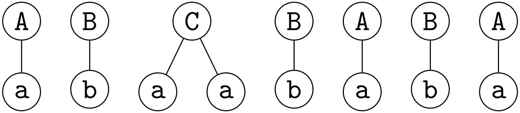


Run-Length Encoding (RLE)

Replace each run with a new symbol: $\underbrace{a \cdots a}_{k \text{ times}} \mapsto (a, k)$.

$$\mathtt{aaabbabaabbbaa} \mapsto (\mathtt{a}, 3)(\mathtt{b}, 2)(\mathtt{a}, 1)(\mathtt{b}, 1)(\mathtt{a}, 2)(\mathtt{b}, 3)(\mathtt{a}, 2)$$

Parse Tree Construction: Compress



RLE

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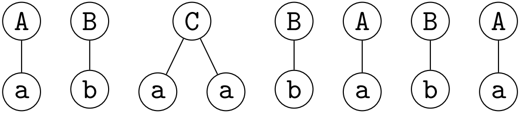

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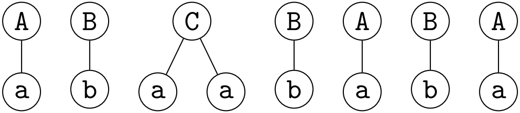

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- Block length: 2–$\mathcal{O}(\log n)$ w.h.p., $\mathcal{O}(1)$ in expectation.
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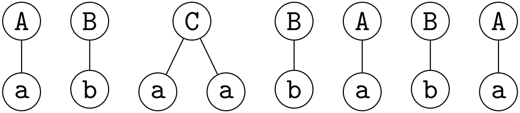

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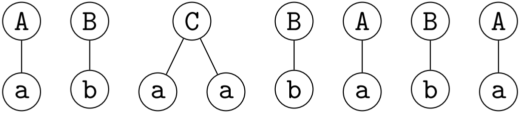

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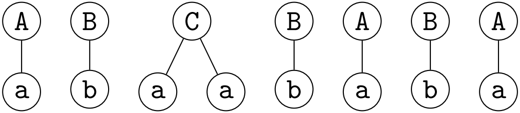

RLE

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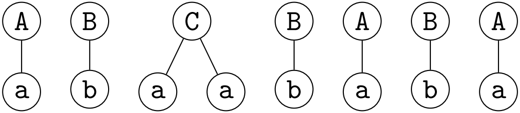

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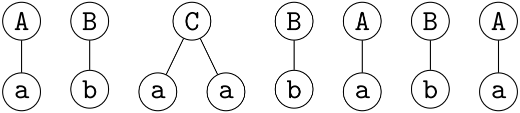

RLE

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How to consistently partition a string without non-trivial runs?

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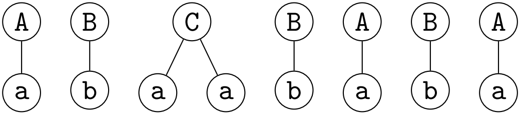

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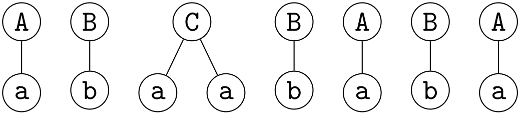
Parse Tree Construction: Compress



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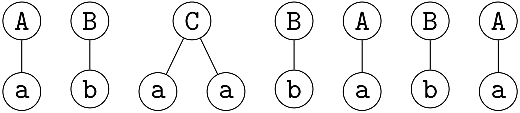

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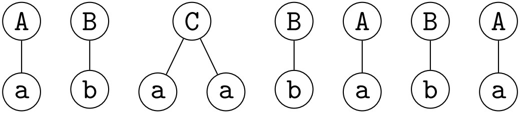

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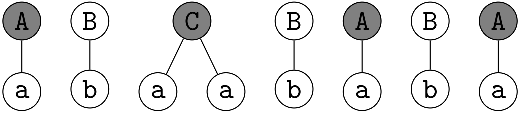

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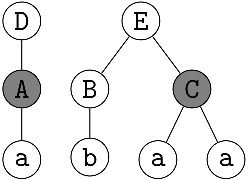

COMPRESS

$\mathtt{D} = (\mathtt{A})$, $\mathtt{E} = (\mathtt{B}, \mathtt{C})$, $\mathtt{F} = (\mathtt{B}, \mathtt{A})$

RLE

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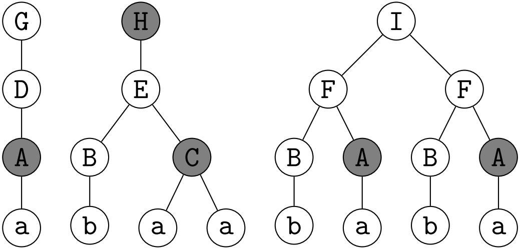
Parse Tree Construction: Example



RLE

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Compress

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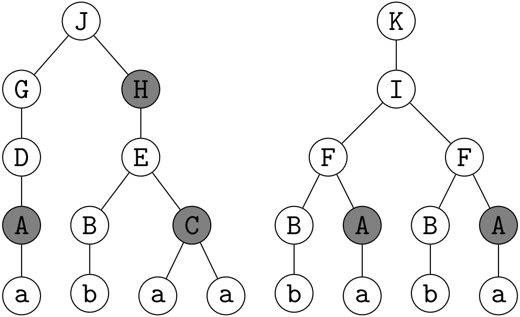

COMPRESS

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RLE

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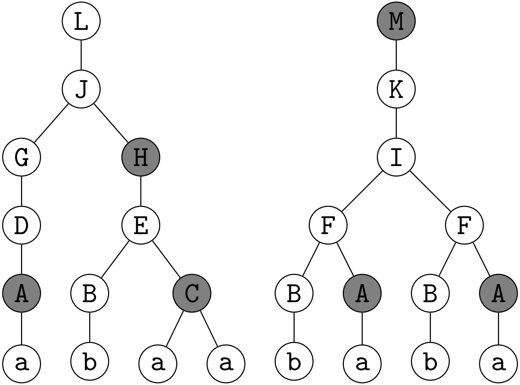

RLE

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COMPRESS

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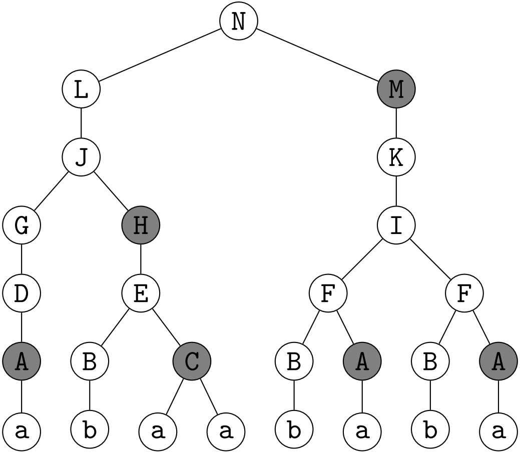

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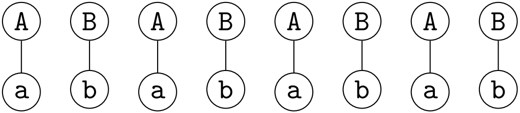
Lemma

For any string w and any $d \in \mathbb{R}_{\geq 0}$:

$$\mathbb{P}[\textsc{Depth}(w) \leq 8(d + \ln|w|)] \geq 1 - e^{-d}$$

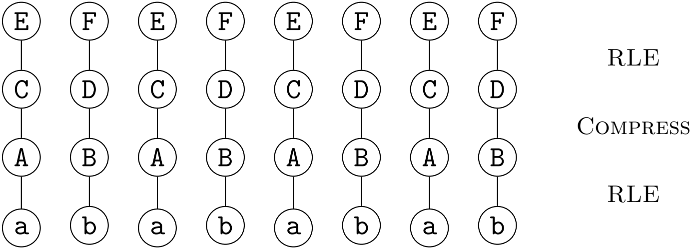
In short: depth $\mathcal{O}(\log n)$ with high probability.

Difficulties



RLE

Difficulties



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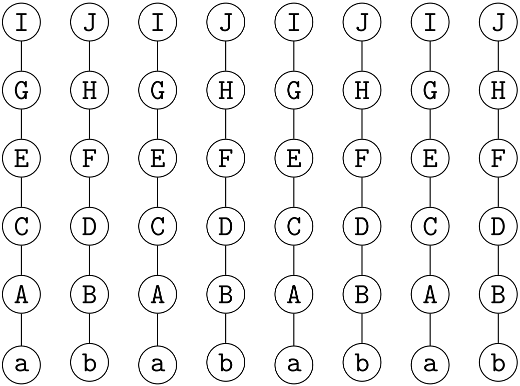

COMPRESS

RLE

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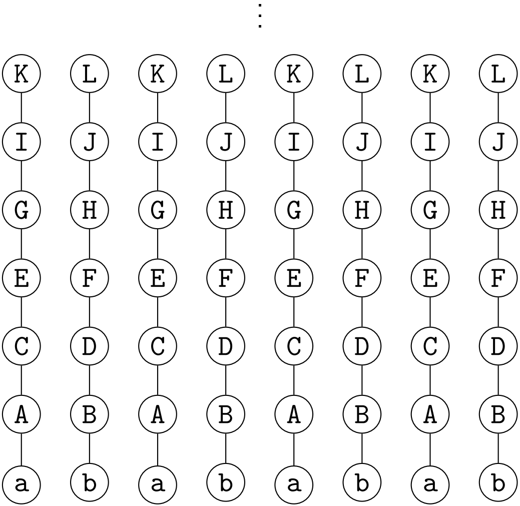

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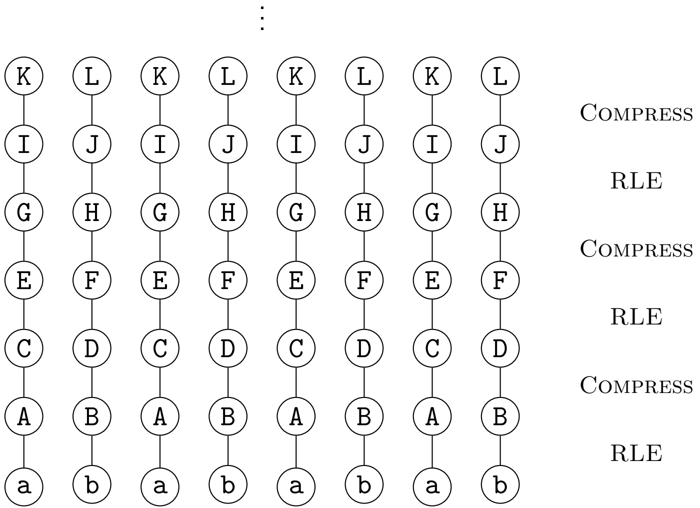
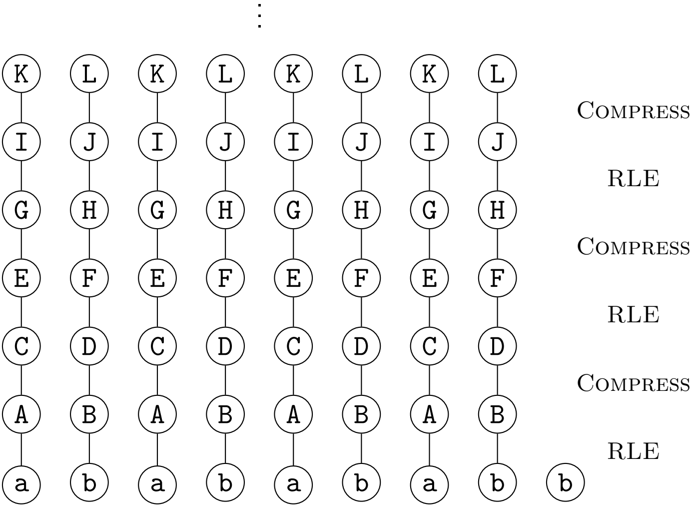


- $\Theta(n \log n)$ nodes with non-negligible probability $\frac{1}{n^{\varepsilon}}$.



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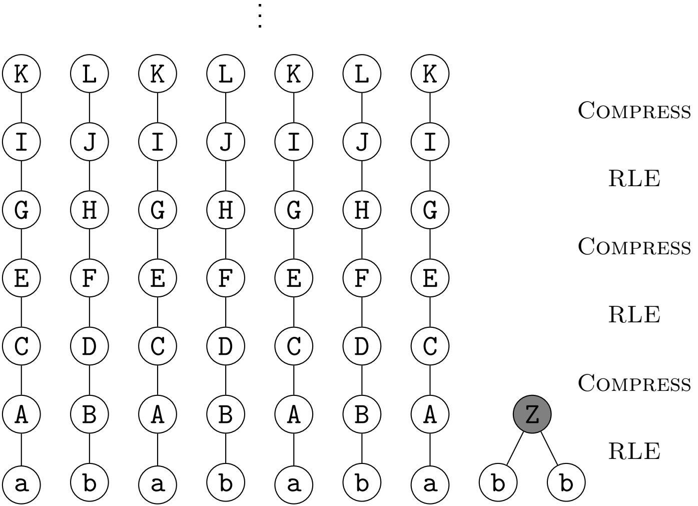

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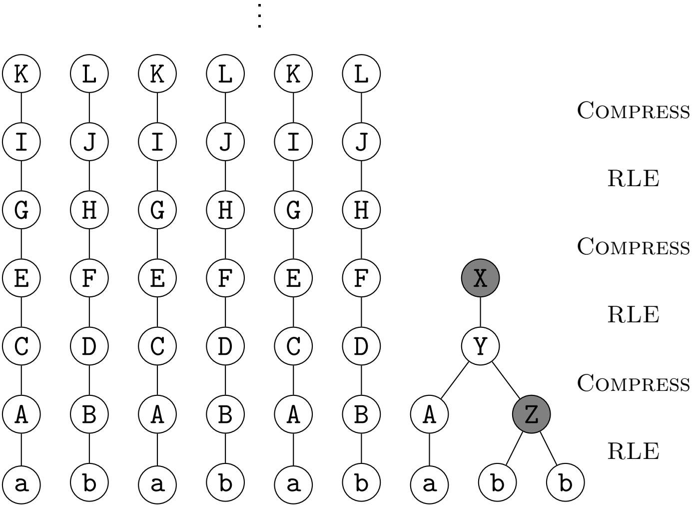

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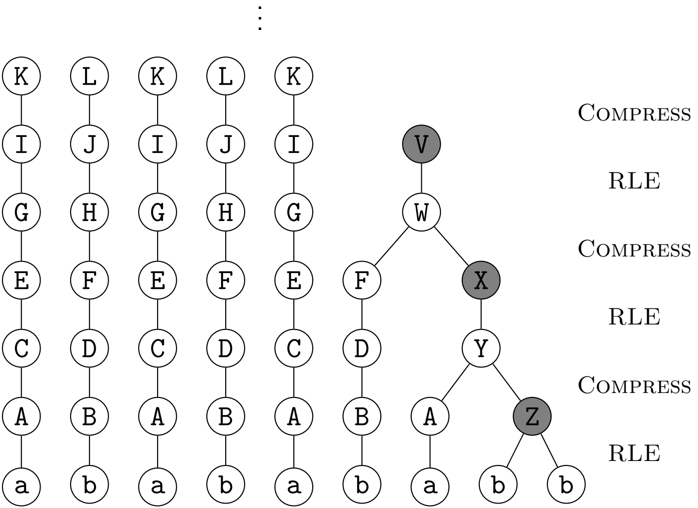

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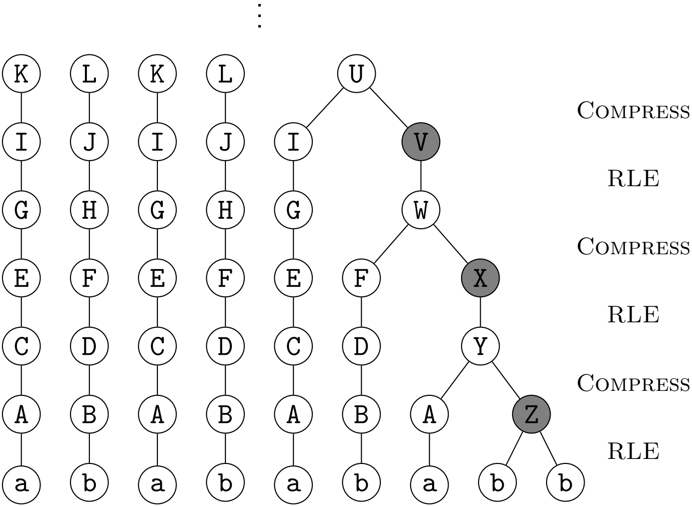

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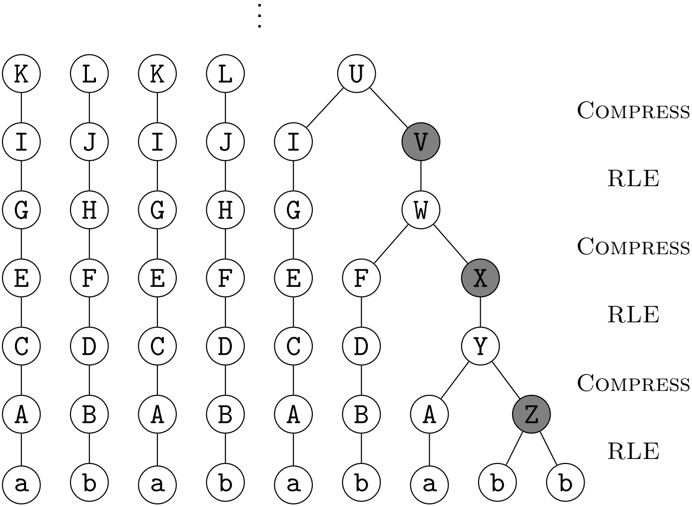
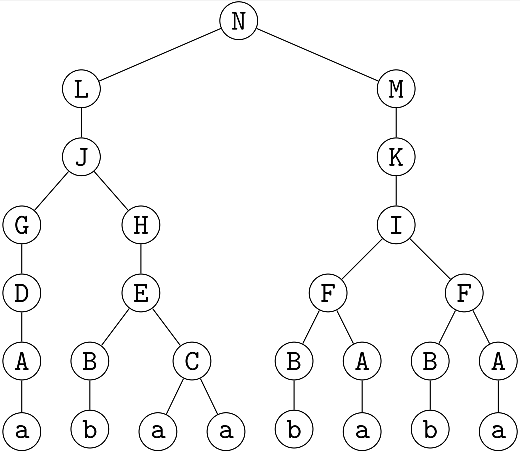


- $\Theta(n \log n)$ nodes with non-negligible probability $\frac{1}{n^{\varepsilon}}$.
- Appending a letter might affect $\Theta(\log^2 n)$ nodes.

Workaround: Compress the Parse Tree



COMPRESS

$\mathtt{N} = (\mathtt{L}, \mathtt{M})$

RLE

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COMPRESS

$\mathtt{J} = (\mathtt{G}, \mathtt{H})$, $\mathtt{K} = (\mathtt{I})$

RLE

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COMPRESS

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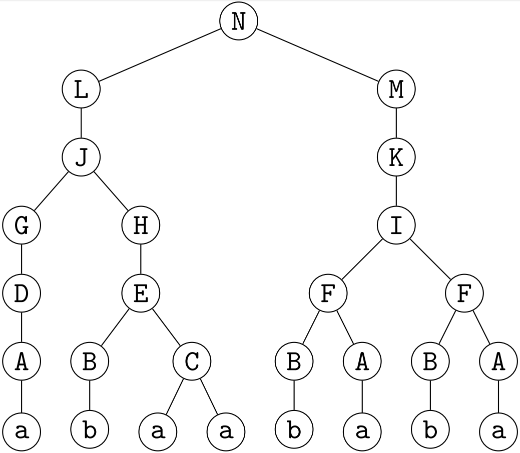

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- Do not replace S with $(S, 1)$ in RLE.

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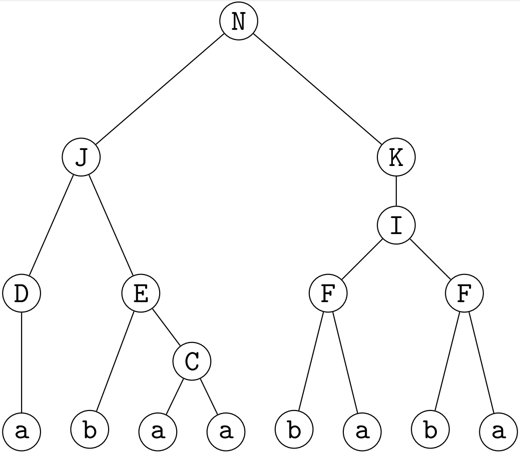

COMPRESS

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RLE

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RLE

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COMPRESS

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RLE

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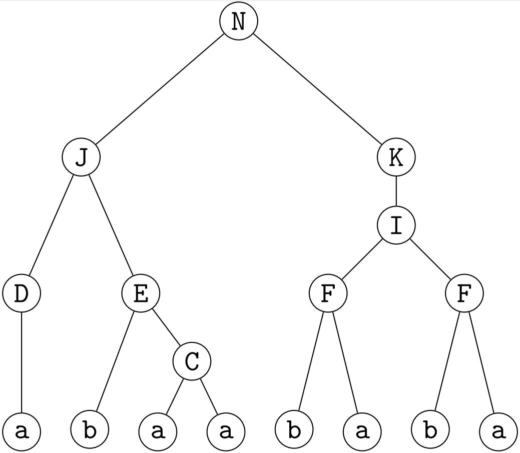

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RLE

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RLE

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COMPRESS

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RLE

$\mathtt{C} = (\mathtt{a}, 2)$

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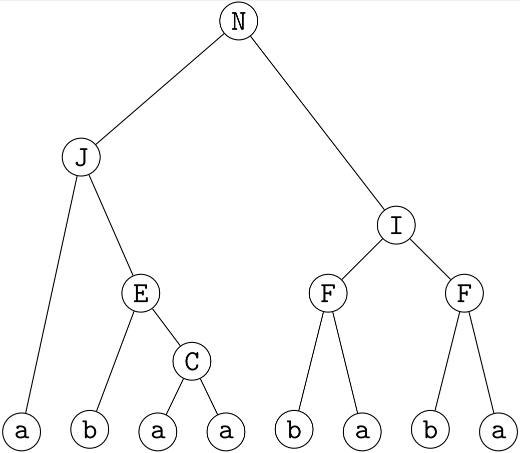
- Do not introduce new symbols for unary blocks in COMPRESS.

Workaround: Compress the Parse Tree



COMPRESS

$\mathtt{N} = (\mathtt{J}, \mathtt{I})$

RLE

COMPRESS

$\mathtt{J} = (\mathtt{a}, \mathtt{E})$

RLE

$\mathtt{I} = (\mathtt{F}, 2)$

COMPRESS

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RLE

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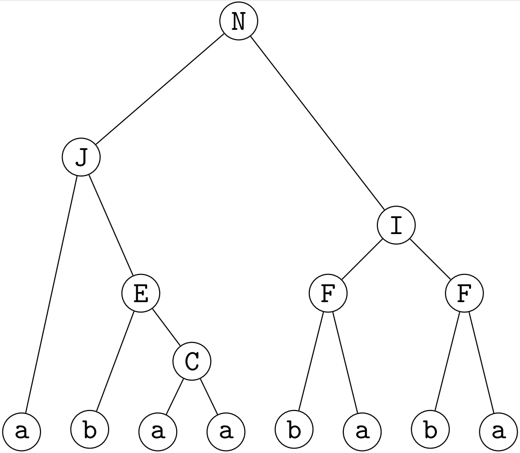


6 COMPRESS

$N = (J, I)$

5 RLE

4 COMPRESS

$J = (a, E)$

3 RLE

$I = (F, 2)$

2 COMPRESS

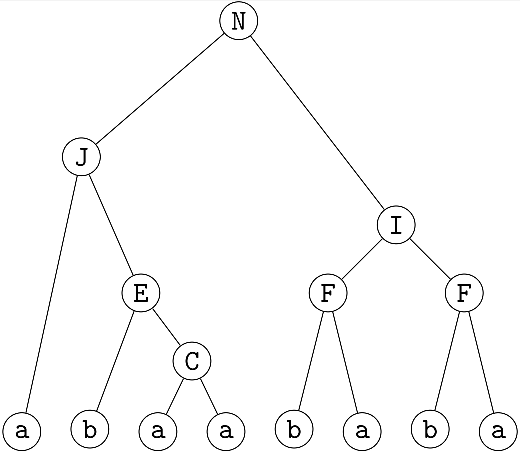
$E = (b, C)$, $F = (b, a)$

1 RLE

$C = (a, 2)$

0

- Do not replace S with $(S, 1)$ in RLE.
- Do not introduce new symbols for unary blocks in COMPRESS.
- Each symbol has a level where it appears ...



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4 COMPRESS

$\mathtt{J} = (\mathtt{a}, \mathtt{E})$

3 RLE

$\mathtt{I} = (\mathtt{F}, 2)$

2 COMPRESS

$\mathtt{E} = (\mathtt{b}, \mathtt{C})$, $\mathtt{F} = (\mathtt{b}, \mathtt{a})$

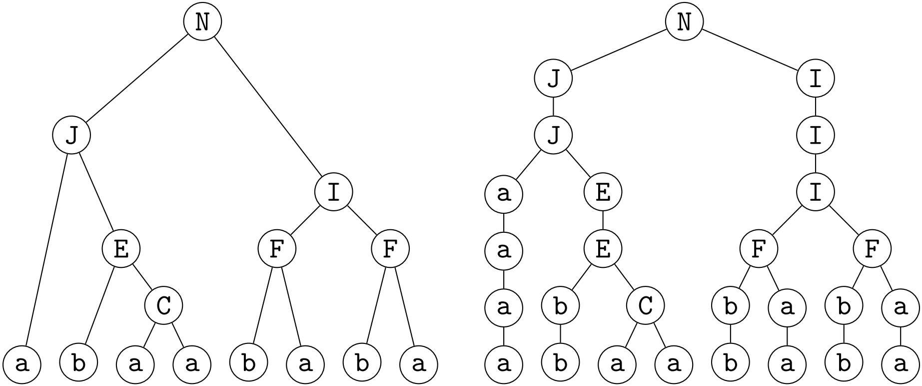
1 RLE

$\mathtt{C} = (\mathtt{a}, 2)$

0

- Do not replace S with $(S, 1)$ in RLE.
- Do not introduce new symbols for unary blocks in COMPRESS.
- Each symbol has a level where it appears ...
- and a random bit for each larger even level ($\mathcal{O}(\log n)$ w.h.p.).

Navigating Uncompressed Parse Trees



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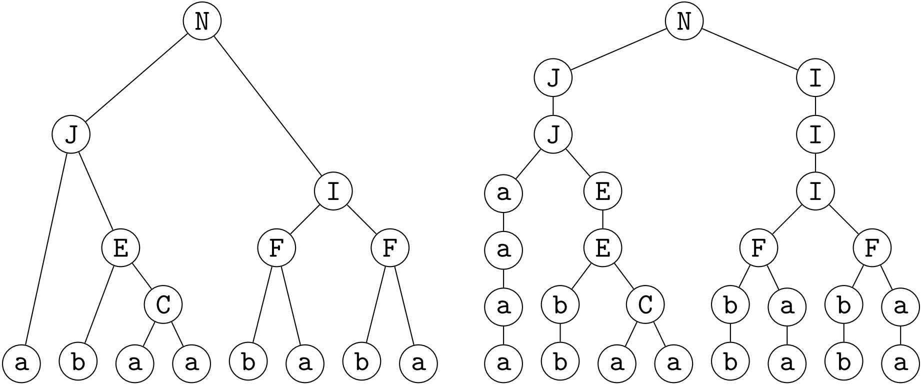

Navigate the uncompressed parse trees in $\mathcal{O}(1)$ time:

- traverse edges: go to the parent or to the k-th child
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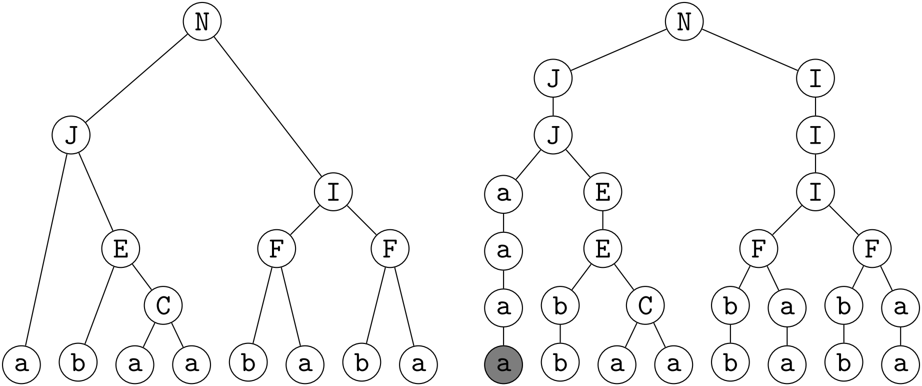

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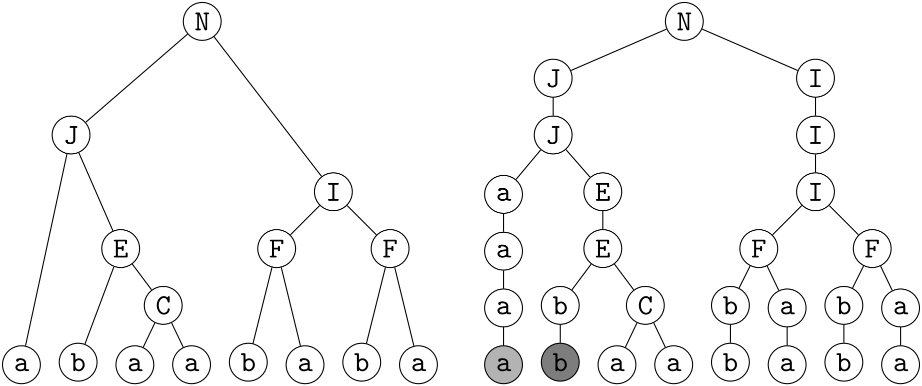

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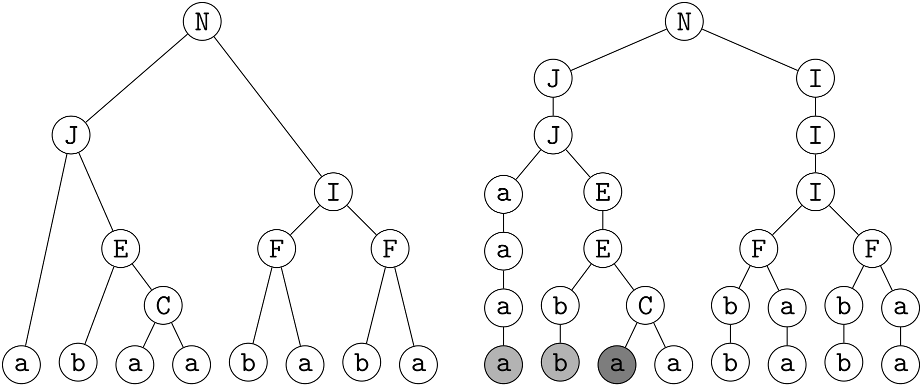

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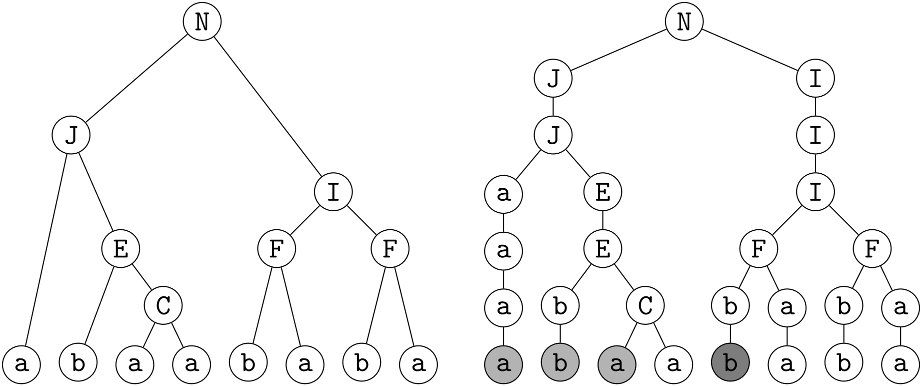

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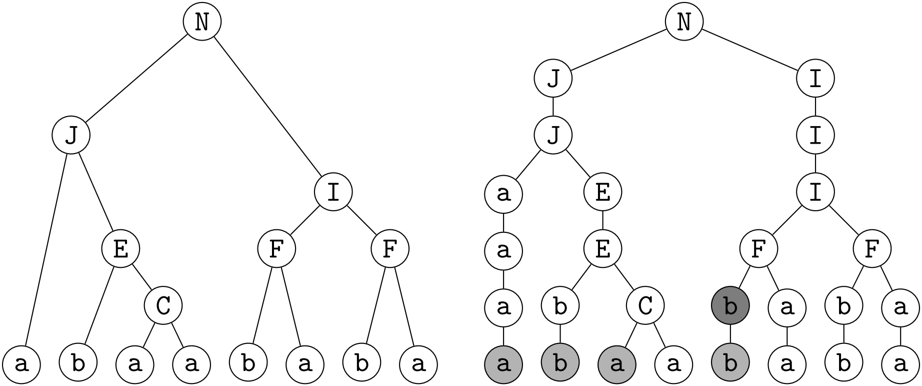

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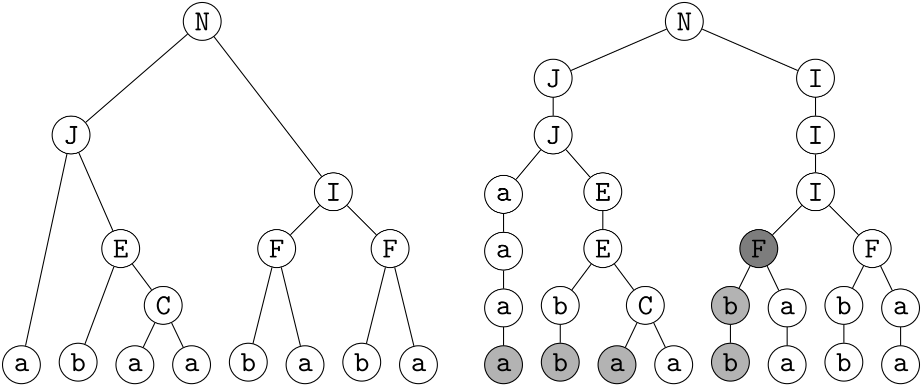

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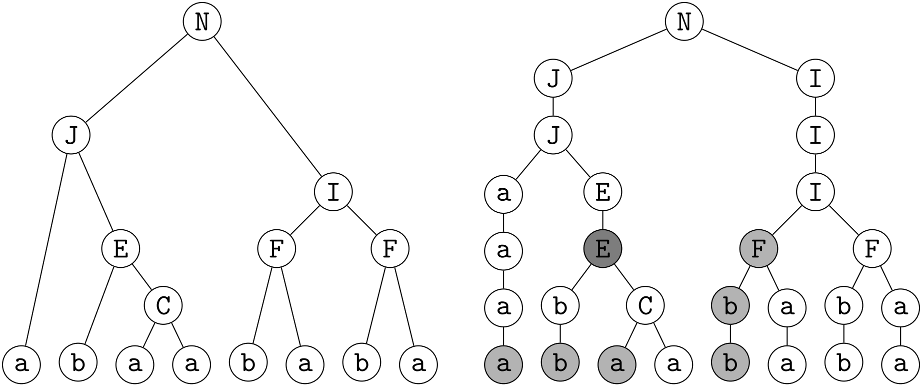

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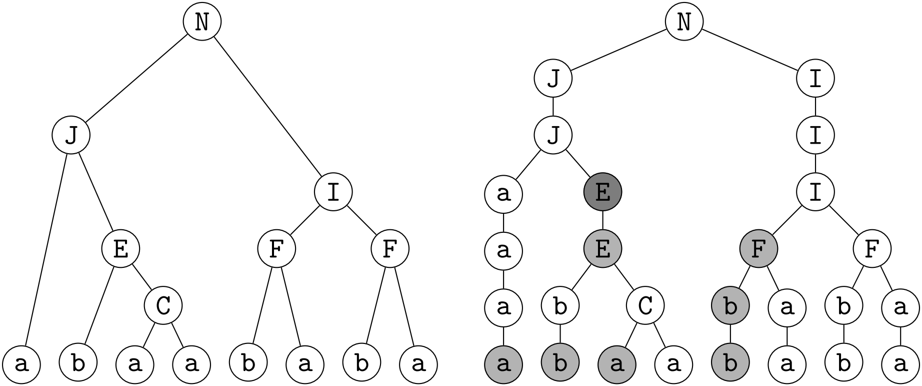

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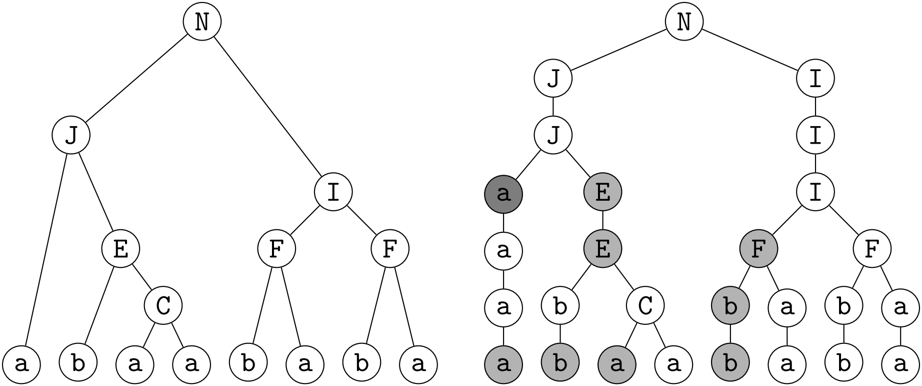

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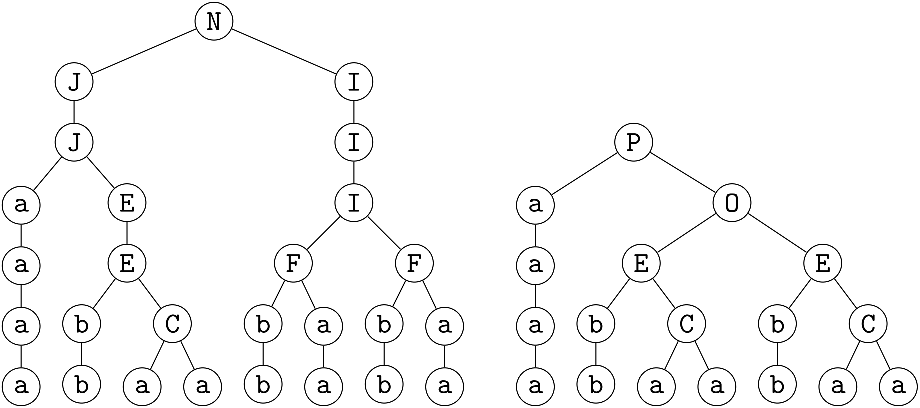

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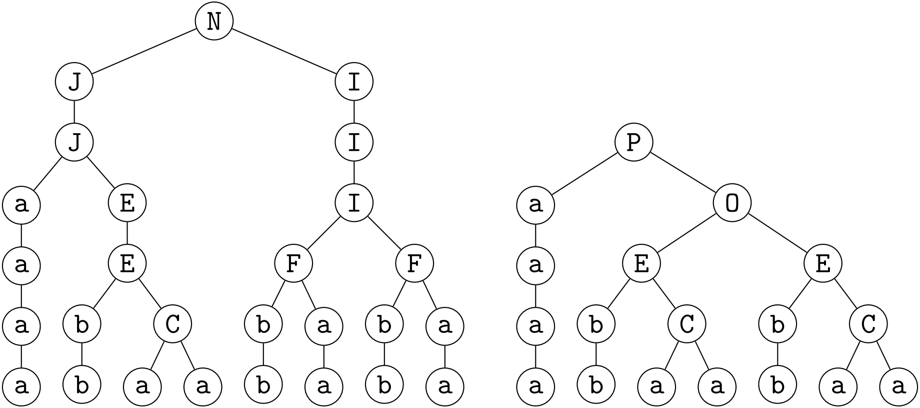

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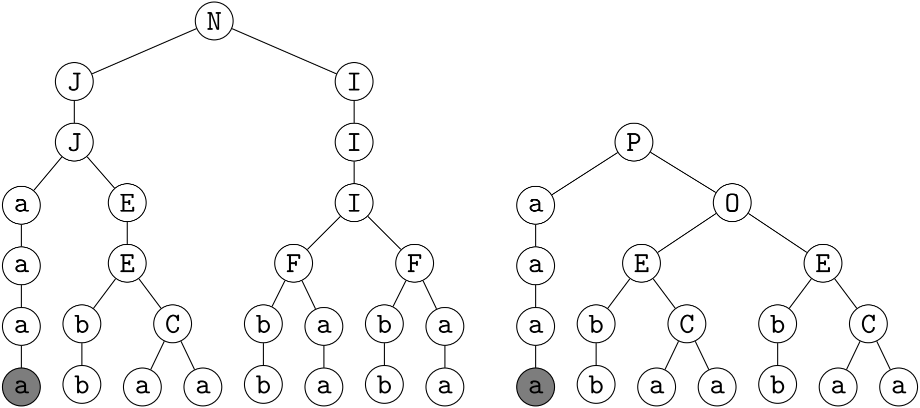

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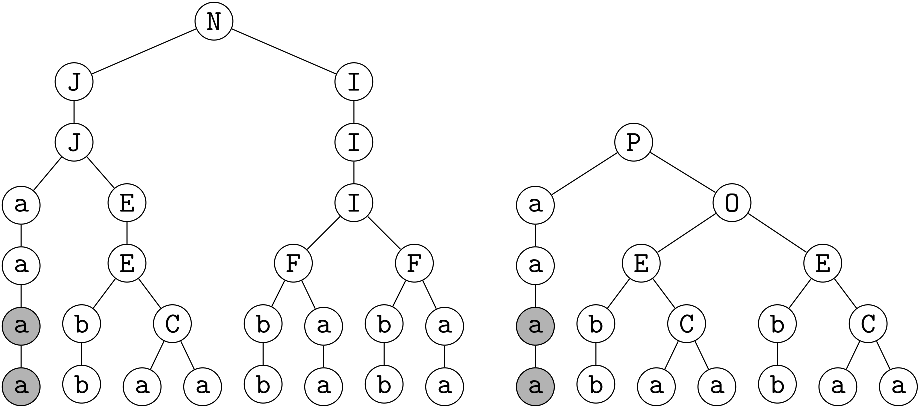

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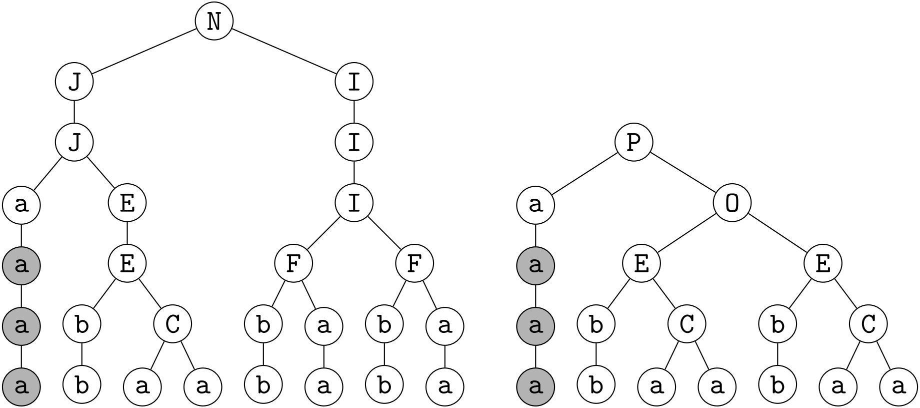

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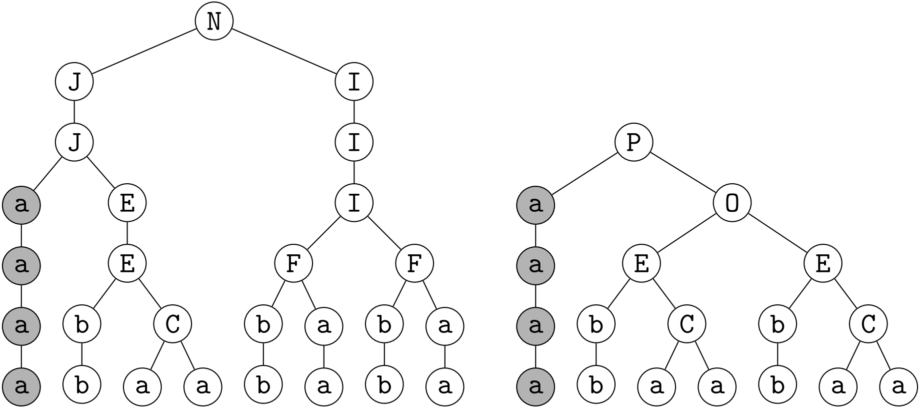

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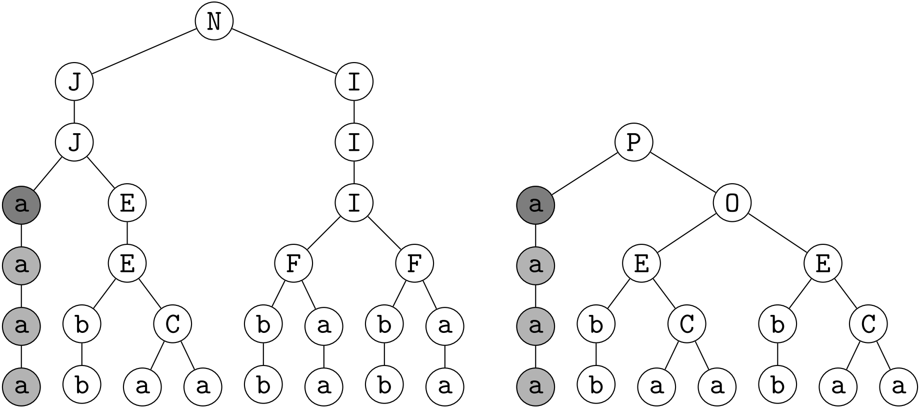

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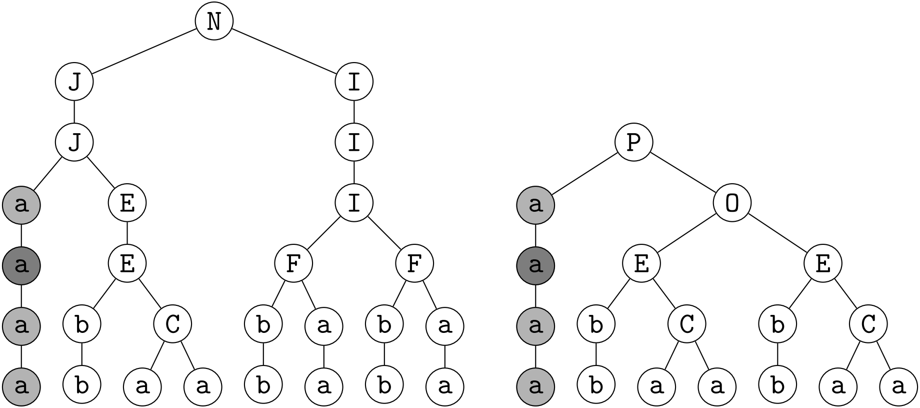

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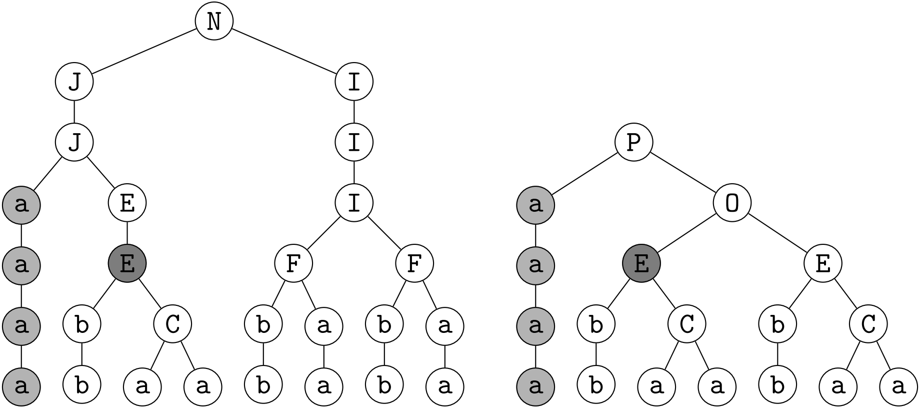

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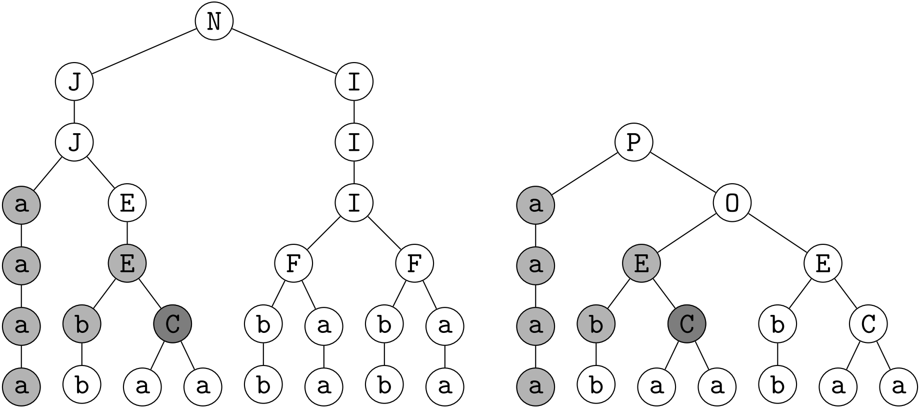

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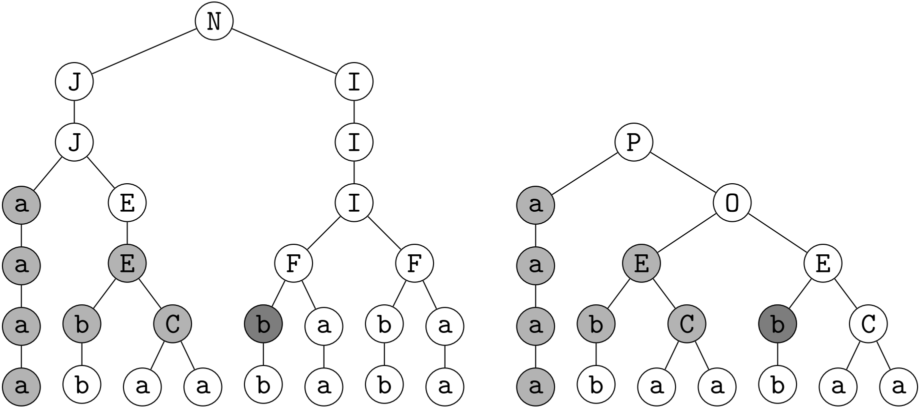

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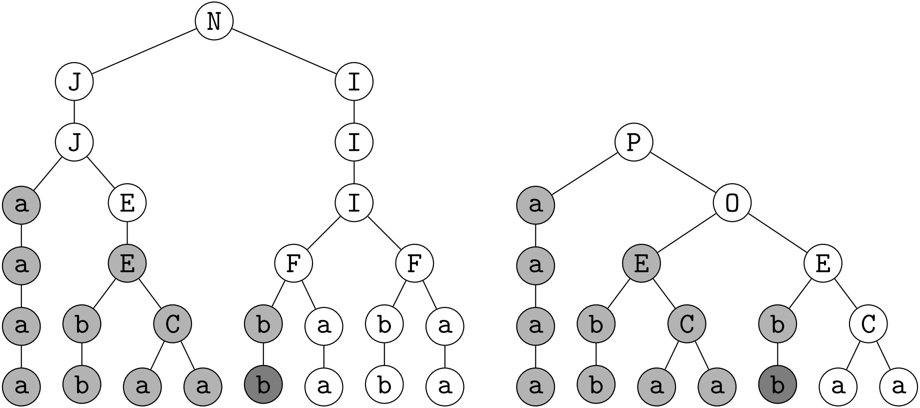

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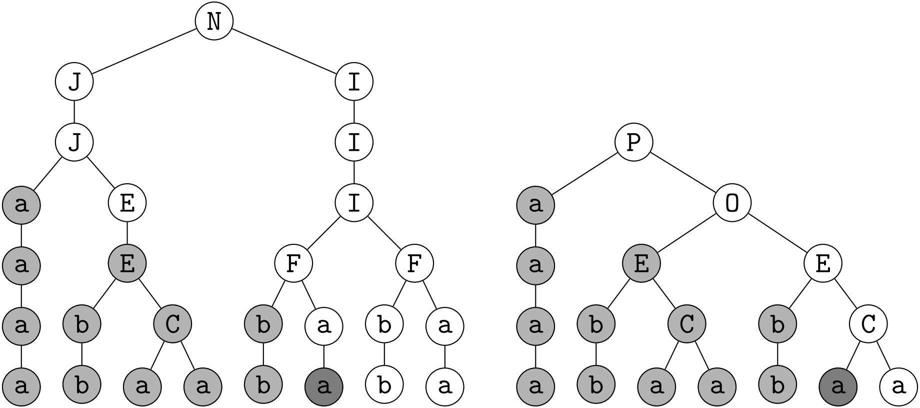

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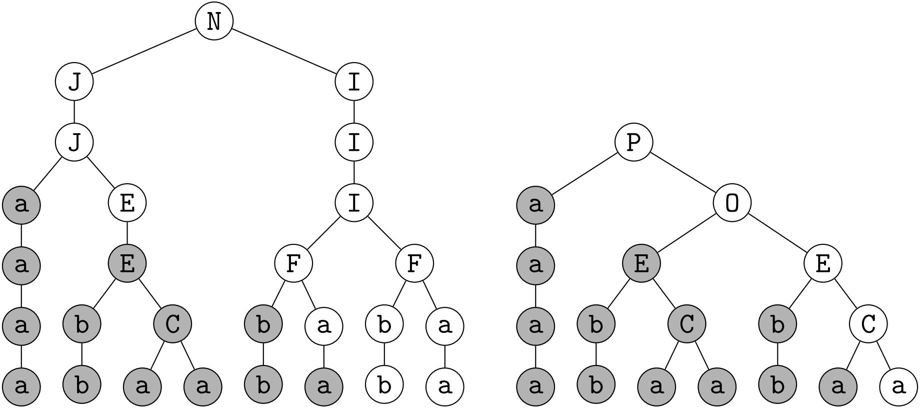

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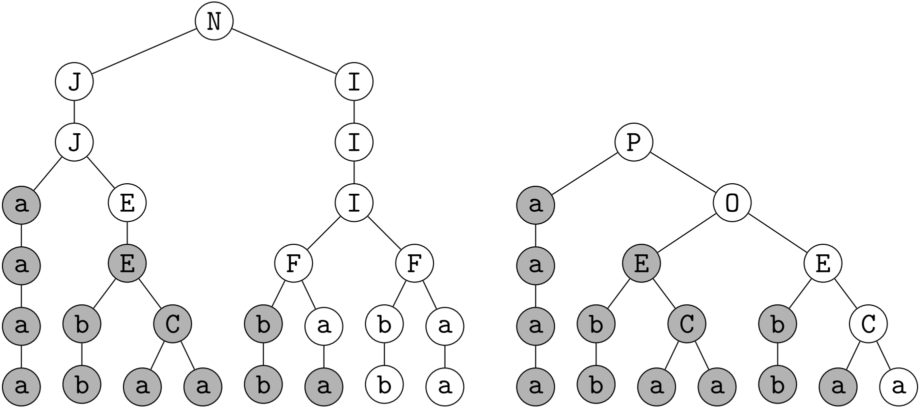

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Total running time: $\mathcal{O}(\min(\textsc{Depth}(w_1), \textsc{Depth}(w_2)))$.

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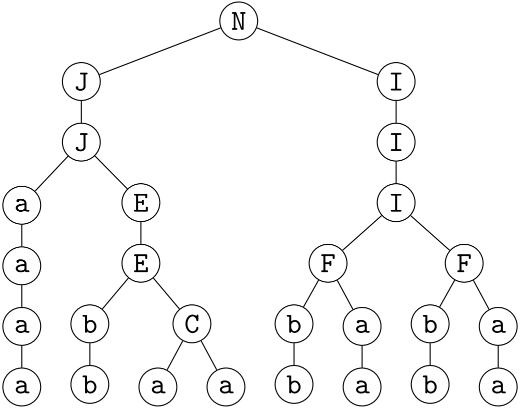
Some nodes in the parse tree are preserved in any context.



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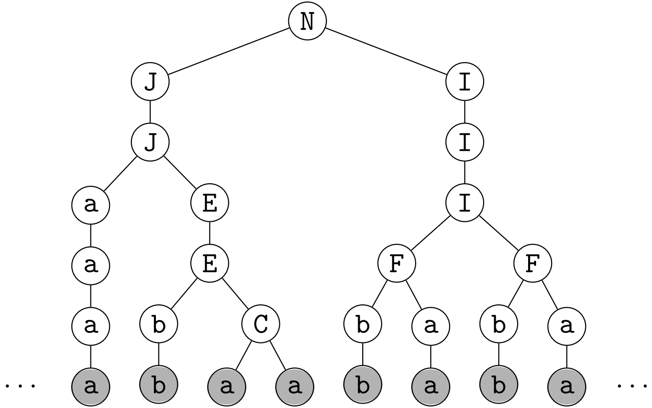

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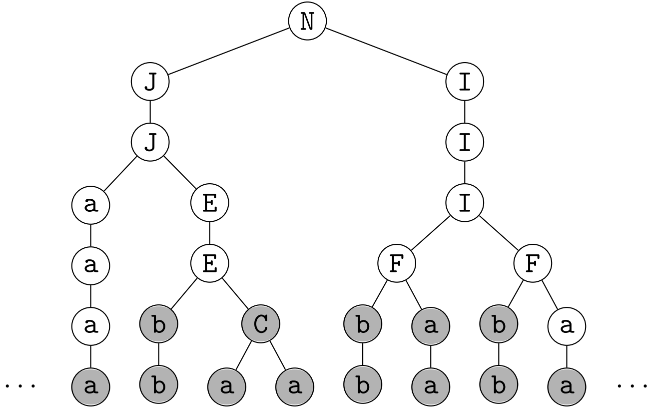

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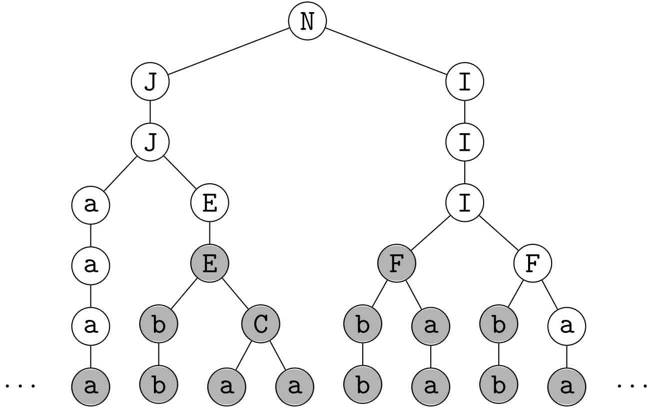

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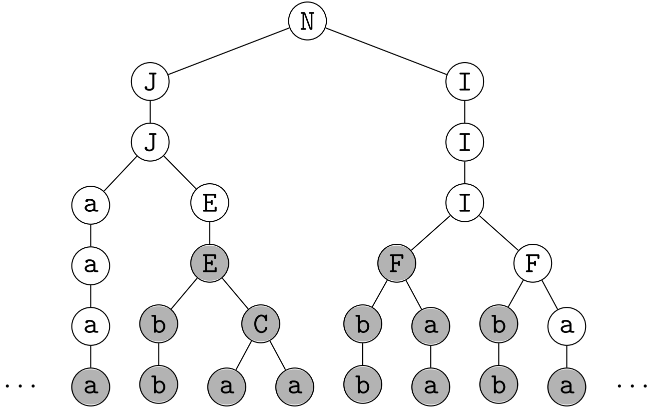

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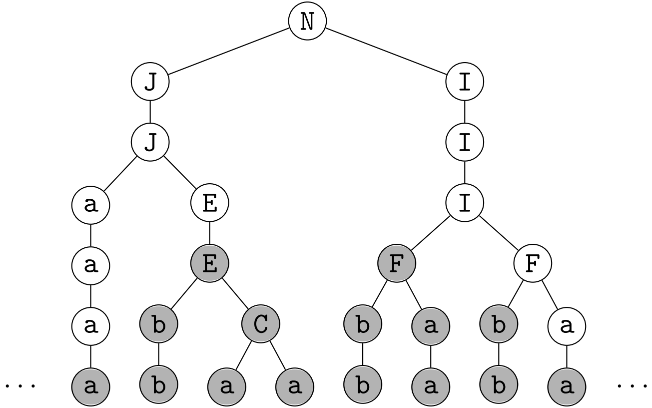

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In general, $|\mathrm{RLE}(D)| \le 2\textsc{Depth}(w)$.
$\mathcal{O}(\textsc{Depth}(w))$ branching context-sensitive nodes.

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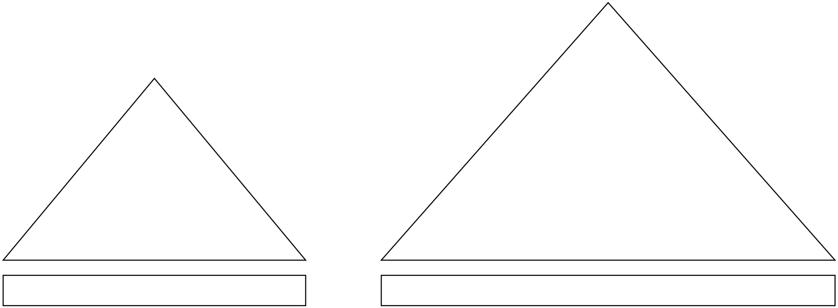

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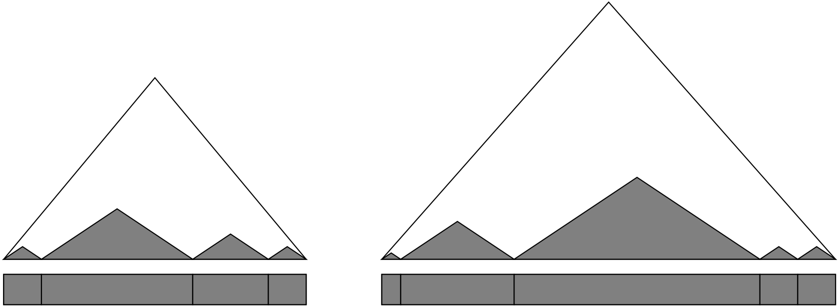

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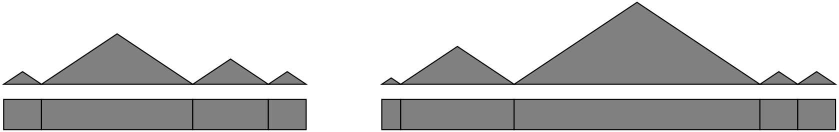

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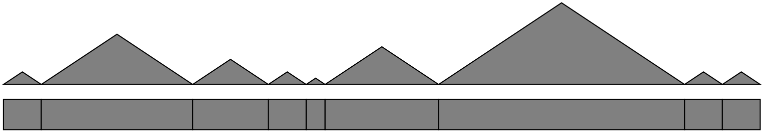

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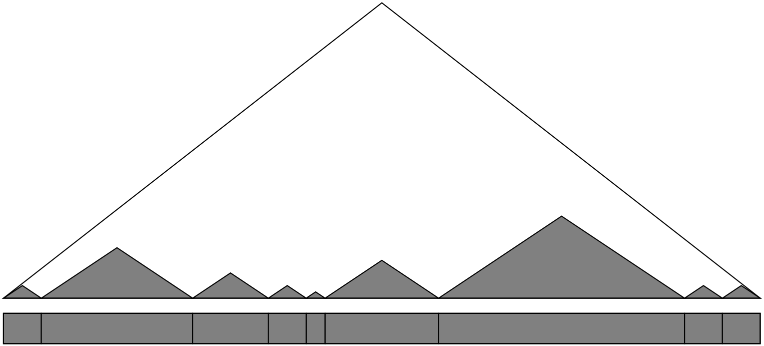

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