Covering Problems for Partial Words and for Indeterminate Strings

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Partial words

$\texttt{b b} \diamondsuit \texttt{ a b b} \diamondsuit \texttt{ a b b} \diamondsuit \texttt{ b a b b b} \diamondsuit \diamondsuit$

- $\diamondsuit$ — *don't care* symbol (*hole*)
- $\diamondsuit \approx c$ for every $c \in \Sigma \cup \{\diamondsuit\}$

Partial words

$$
\begin{array}{cccccccccccccccccc}
\mathtt{b} & \mathtt{b} & \Diamond & \mathtt{a} & \Diamond \\
 & & \wr\wr \\
\mathtt{b} & \mathtt{b} & \Diamond & \mathtt{a} & \mathtt{b} & \mathtt{b} & \Diamond & \mathtt{a} & \mathtt{b} & \mathtt{b} & \Diamond & \mathtt{b} & \mathtt{a} & \mathtt{b} & \mathtt{b} & \mathtt{b} & \Diamond & \Diamond
\end{array}
$$

- $\Diamond$ — *don't care* symbol (*hole*)
- $\Diamond \approx c$ for every $c \in \Sigma \cup \{\Diamond\}$

Partial words

$$
\begin{array}{cccccccccccccccccc}
 & b & b & \lozenge & a & \lozenge & & & & & & & & & & & & \\
 & & & \not\approx & & & & & & & & & & & & & & \\
b & b & \lozenge & a & b & b & \lozenge & a & b & b & \lozenge & b & a & b & b & b & \lozenge & \lozenge
\end{array}
$$

- $\lozenge$ — *don't care* symbol (*hole*)
- $\lozenge \approx c$ for every $c \in \Sigma \cup \{\lozenge\}$

Indeterminate strings

a	b	c	d	a	c	a	b	c	a	b	a	c	a	b	a	b	c
	c			b		d			c		c	d		c			d
									d								

- Symbols — non-empty subsets of Σ

Indeterminate strings

a	b	c	d	a	c	a	b	c	a	b	a	c	a	b	a	b	c
	c			b		d			c		c	d		c			d
									d								

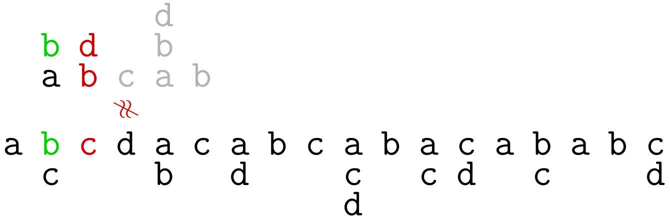
- Symbols — non-empty subsets of Σ
- Non-solid symbols — subsets of size at least 2

Indeterminate strings

			d														
b	d		b														
a	b	c	a	b													
		$\approx$															
a	b	c	d	a	c	a	b	c	a	b	a	c	a	b	a	b	c
	c			b		d			c		c	d		c			d
									d								

- Symbols — non-empty subsets of Σ
- Non-solid symbols — subsets of size at least 2
- $A \approx B$ if $A \cap B \neq \emptyset$

Indeterminate strings



- Symbols — non-empty subsets of Σ
- Non-solid symbols — subsets of size at least 2
- $A \approx B$ if $A \cap B \neq \emptyset$

Covering a solid string

Definition (Apostolico, Farach, Iliopoulos; 1991)

A string S is a *cover* of a string T if each position in T lies within an occurrence of S.

```
                                a a b a a
                           a a b a a
                   a a b a a
          a a b a a
a a b a a

a a b a a a b a a b a a a b a a a a b a a
```

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```
                          a a b a a
                     a a b a a
                 a a b a a
              a a b a a
          a a b a a
          a a b a a a b a a b a a a b a a a a b a a
```

- $S =$ aabaa is a cover.

Covering a solid string

Definition (Apostolico, Farach, Iliopoulos; 1991)

A string S is a *cover* of a string T if each position in T lies within an occurrence of S.

```
                                a a b a
                      a a b a
              a a b a
        a a b a
a a b a

a a b a a a b a a b a a a b a a a a b a a
```

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```
                                a a b a
                        a a b a
                a a b a
          a a b a
    a a b a
a a b a a a b a a b a a a b a a a a b a a
```

- $S =$ aaba is not a cover.

Results for solid strings

Linear-time algorithms for natural problems:

1991; Apostolico et al. shortest cover

1992; Breslauer shortest cover (online)

1994; Moore & Smyth all covers

2002; Li & Smyth all covers (online)

Results for solid strings

Linear-time algorithms for natural problems:

1991; Apostolico et al. shortest cover

1992; Breslauer shortest cover (online)

1994; Moore & Smyth all covers

2002; Li & Smyth all covers (online)

Numerous generalizations:

1992; Iliopoulos & Smyth k-covers

1993; Iliopoulos et al. seeds

2002; Sim et al. approximate covers

2012; Flouri et al. enhanced covers

2013; K. et al. partial covers

Covering partial words and indeterminate strings

Definition

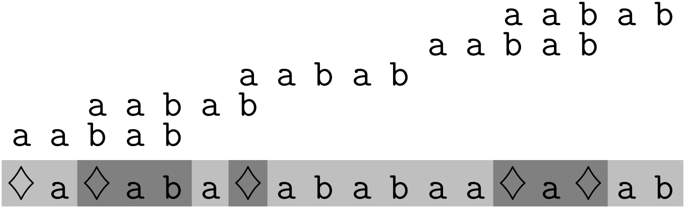
A *solid* string S is a cover of an *indeterminate* string T if each position in T lies within an occurrence of S.

```
                          a a b a b
                      a a b a b
            a a b a b
    a a b a b
a a b a b
◇ a ◇ a b a ◇ a b a b a a ◇ a ◇ a b
```

Covering partial words and indeterminate strings

Definition

A *solid* string S is a cover of an *indeterminate* string T if each position in T lies within an occurrence of S.



- $S =$ aabab is a cover of T.

Results for partial words and indeterminate strings

Problem

Find a shortest cover of a given indeterminate string T.

- n length of T
- k number of non-solid symbols in T
- σ alphabet size ($\sigma \leq n$)

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Our results:

- NP-hardness, already for binary partial words,

Results for partial words and indeterminate strings

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- $\mathcal{O}(nk^4 + 2^{\mathcal{O}(k \log k)})$-time algorithm,

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- $\mathcal{O}(nk^4 + 2^{\mathcal{O}(\sqrt{k} \log k)})$-time version for partial words,

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- $2^{o(\sqrt{k})} n^{\mathcal{O}(1)}$ lower bound already for binary partial words (under ETH).

Results for partial words and indeterminate strings

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- $\mathcal{O}(\cancel{nk^4} n^{\mathcal{O}(1)} + 2^{\mathcal{O}(\sqrt{k} \log k)})$-time version for partial words,
- $2^{o(\sqrt{k})} n^{\mathcal{O}(1)}$ lower bound already for binary partial words (under ETH).

How to guess a cover? (in typical cases)

- Checking whether a solid string S is a cover is easy.
- We can afford verifying polynomially many candidates.

```
        b b b a b         b b b a b
b b b a b         b b b a b
b b b a b b ◇ a b b b b a b b b ◇ ◇
```

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b b b a b         b b b a b
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```

Attempts:

1. Guess $\ell = |S|$. Succeed if $T[1..\ell]$ is solid.

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```
            b b b a b             b b b a b
b b b a b             b b b a b
```

b b b a b b $\diamondsuit$ a b b b b a b b b $\diamondsuit$ $\diamondsuit$

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b b b a b         b b b a b
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```

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1. Guess $\ell = |S|$. Succeed if $T[1..\ell]$ is solid.
2. Guess $\ell = |S|$ and $i \in Occ(S, T)$. Succeed if $T[i..i + \ell - 1]$ is solid.

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b b b a b   b b b a b

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```
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    b b b a b                 b b b a b
    b b ◇ a b b ◇ a b b ◇ b a b b b ◇ ◇
```

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b b b a b   b b b a b

b $\Diamond$ b a b   b b $\Diamond$ a b

b b $\Diamond$ a b b $\Diamond$ a b b $\Diamond$ b a b b b $\Diamond$ $\Diamond$

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```
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b b ◇ a b b ◇ a b b ◇ b a b b b ◇ ◇
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Which covers could we miss?

```
    a a b a b         a a b a b
a a b a b   a a b a b     a a b a b
◇ a ◇ a b a ◇ a b a b a a ◇ a ◇ a b
```

Which covers could we miss?

```
    ◇ a ◇ a b         a a b a b
◇ a b a ◇   a a b a b     a a b a b
◇ a ◇ a b a ◇ a b a b a a ◇ a ◇ a b
```

Which covers could we miss?

```
        a a b a b               a a b a b
◇ a b a b     ◇ a ◇ a b               a a b a b
◇ a ◇ a b a ◇ a b a b a a ◇ a ◇ a b
```

Which covers could we miss?

a a b a b ◇ a ◇ a b

a a ◇ a ◇ a a b a b a a b a b

◇ a ◇ a b a ◇ a b a b a a ◇ a ◇ a b

Which covers could we miss?

```
    a a b a b         a a b a b
◇ a ◇ a b   a a b a b   ◇ a ◇ a b
◇ a ◇ a b a ◇ a b a b a a ◇ a ◇ a b
```

Which covers could we miss?

$\diamondsuit$ a $\diamondsuit$ a b a $\diamondsuit$ a b a b a a $\diamondsuit$ a $\diamondsuit$ a b

- If S was not found, then for each $i \in Occ(S,T)$ there exists j such that $T[1+j] = T[i+j] = \diamondsuit$.

Which covers could we miss?

$\diamondsuit$ a $\diamondsuit$ a b a $\diamondsuit$ a b a b a a $\diamondsuit$ a $\diamondsuit$ a b

$\diamondsuit$ a $\diamondsuit$ a b a $\diamondsuit$ a b a b a a $\diamondsuit$ a $\diamondsuit$ a b

- If S was not found, then for each $i \in Occ(S,T)$ there exists j such that $T[1+j] = T[i+j] = \diamondsuit$.
- Here *ambiguous* positions ($\mathcal{A}$) are: 1

Which covers could we miss?

$\diamondsuit$ a $\diamondsuit$ a b a $\diamondsuit$ a b a b a a $\diamondsuit$ a $\diamondsuit$

$\diamondsuit$ a $\diamondsuit$ a b a $\diamondsuit$ a b a b a a $\diamondsuit$ a $\diamondsuit$ a b

- If S was not found, then for each $i \in Occ(S, T)$ there exists j such that $T[1+j] = T[i+j] = \diamondsuit$.
- Here *ambiguous* positions ($\mathcal{A}$) are: 1, 3

Which covers could we miss?

$\diamondsuit$ a $\diamondsuit$ a b a $\diamondsuit$ a b a b a a $\diamondsuit$

$\diamondsuit$ a $\diamondsuit$ a b a $\diamondsuit$ a b a b a a $\diamondsuit$ a $\diamondsuit$ a b

- If S was not found, then for each $i \in Occ(S, T)$ there exists j such that $T[1 + j] = T[i + j] = \diamondsuit$.
- Here *ambiguous* positions ($\mathcal{A}$) are: 1, 3, 5

Which covers could we miss?

$$\begin{array}{cccccccccccccccccc} & & & & & & \diamondsuit & a & \diamondsuit & a & b & a & \diamondsuit & a & b & a & b & a \\ \diamondsuit & a & \diamondsuit & a & b & a & \diamondsuit & a & b & a & b & a & a & \diamondsuit & a & \diamondsuit & a & b \end{array}$$

- If S was not found, then for each $i \in Occ(S, T)$ there exists j such that $T[1 + j] = T[i + j] = \diamondsuit$.
- Here *ambiguous* positions ($\mathcal{A}$) are: 1, 3, 5, 7

Which covers could we miss?

$$\diamondsuit\ \mathtt{a}\ \diamondsuit\ \mathtt{a}\ \mathtt{b}\ \mathtt{a}\ \diamondsuit\ \mathtt{a}\ \mathtt{b}\ \mathtt{a}\ \mathtt{b}$$

$$\diamondsuit\ \mathtt{a}\ \diamondsuit\ \mathtt{a}\ \mathtt{b}\ \mathtt{a}\ \diamondsuit\ \mathtt{a}\ \mathtt{b}\ \mathtt{a}\ \mathtt{b}\ \mathtt{a}\ \mathtt{a}\ \diamondsuit\ \mathtt{a}\ \diamondsuit\ \mathtt{a}\ \mathtt{b}$$

- If S was not found, then for each $i \in Occ(S,T)$ there exists j such that $T[1+j] = T[i+j] = \diamondsuit$.
- Here *ambiguous* positions ($\mathcal{A}$) are: 1, 3, 5, 7, 8

Which covers could we miss?

$\diamondsuit$ a $\diamondsuit$ a b a $\diamondsuit$ a b a b

$\diamondsuit$ a $\diamondsuit$ a b a $\diamondsuit$ a b a b a a $\diamondsuit$ a $\diamondsuit$ a b

- If S was not found, then for each $i \in Occ(S, T)$ there exists j such that $T[1+j] = T[i+j] = \diamondsuit$. Moreover, $T[1..j] \approx T[i..i+j-1]$.
- Here *ambiguous* positions ($\mathcal{A}$) are: 1, 3, 5, 7, 8

Which covers could we miss?

$$\begin{array}{cccccccccccccccccc} & & & & & & & & & \diamondsuit & \mathtt{a} & \diamondsuit & \mathtt{a} & \mathtt{b} & \mathtt{a} & \diamondsuit & \mathtt{a} & \mathtt{b} \\ \diamondsuit & \mathtt{a} & \diamondsuit & \mathtt{a} & \mathtt{b} & \mathtt{a} & \diamondsuit & \mathtt{a} & \mathtt{b} & \mathtt{a} & \mathtt{b} & \mathtt{a} & \mathtt{a} & \diamondsuit & \mathtt{a} & \diamondsuit & \mathtt{a} & \mathtt{b} \end{array}$$

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- Here *ambiguous* positions ($\mathcal{A}$) are: 1, 3, 5, 7, 8, 10

Which covers could we miss?

$$\diamondsuit\ \mathtt{a}\ \diamondsuit\ \mathtt{a}\ \mathtt{b}\ \mathtt{a}\ \diamondsuit$$

$$\diamondsuit\ \mathtt{a}\ \diamondsuit\ \mathtt{a}\ \mathtt{b}\ \mathtt{a}\ \diamondsuit\ \mathtt{a}\ \mathtt{b}\ \mathtt{a}\ \mathtt{b}\ \mathtt{a}\ \mathtt{a}\ \diamondsuit\ \mathtt{a}\ \diamondsuit\ \mathtt{a}\ \mathtt{b}$$

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- Here *ambiguous* positions ($\mathcal{A}$) are: 1, 3, 5, 7, 8, 10, 12

Which covers could we miss?

$$\begin{array}{cccccccccccccccccc} & & & & & & & & & & & & \diamondsuit & \texttt{a} & \diamondsuit & \texttt{a} & \texttt{b} \\ \diamondsuit & \texttt{a} & \diamondsuit & \texttt{a} & \texttt{b} & \texttt{a} & \diamondsuit & \texttt{a} & \texttt{b} & \texttt{a} & \texttt{b} & \texttt{a} & \texttt{a} & \diamondsuit & \texttt{a} & \diamondsuit & \texttt{a} & \texttt{b} \end{array}$$

- If S was not found, then for each $i \in Occ(S,T)$ there exists j such that $T[1+j] = T[i+j] = \diamondsuit$. Moreover, $T[1..j] \approx T[i..i+j-1]$.
- Here *ambiguous* positions ($\mathcal{A}$) are: 1, 3, 5, 7, 8, 10, 12, 14

Which covers could we miss?

$$\diamondsuit\ \texttt{a}\ \texttt{a}\ \diamondsuit$$

$$\diamondsuit\ \texttt{a}\ \diamondsuit\ \texttt{a}\ \texttt{b}\ \texttt{a}\ \diamondsuit\ \texttt{a}\ \texttt{b}\ \texttt{a}\ \texttt{b}\ \texttt{a}\ \texttt{a}\ \diamondsuit\ \texttt{a}\ \diamondsuit\ \texttt{a}\ \texttt{b}$$

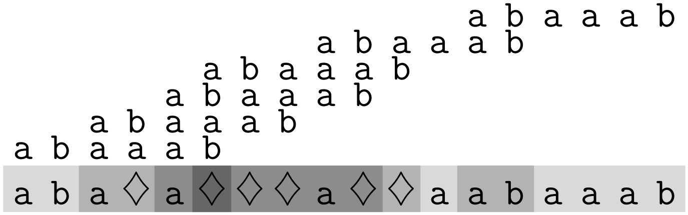
- If S was not found, then for each $i \in Occ(S, T)$ there exists j such that $T[1+j] = T[i+j] = \diamondsuit$. Moreover, $T[1..j] \approx T[i..i+j-1]$.
- Here *ambiguous* positions ($\mathcal{A}$) are: 1, 3, 5, 7, 8, 10, 12, 14, and 16.

Which covers could we miss?

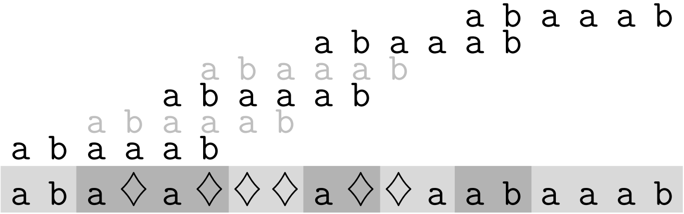
$\Diamond$ a $\Diamond$ a b a $\Diamond$ a b a b a a $\Diamond$ a $\Diamond$ a b

- If S was not found, then for each $i \in Occ(S,T)$ there exists j such that $T[1+j] = T[i+j] = \Diamond$.
  Moreover, $T[1..j] \approx T[i..i+j-1]$.
- Here *ambiguous* positions ($\mathcal{A}$) are: 1, 3, 5, 7, 8, 10, 12, 14, and 16.
  In general, there are at most $1 + \frac{k(k-1)}{2} = \mathcal{O}(k^2)$ ambiguous positions.

Covering set of a cover



Covering set of a cover



Definition

A set $\mathcal{P} \subseteq Occ(S, T)$ is a *covering set* for S if every position in T is covered by at least one occurrence of S in $\mathcal{P}$.

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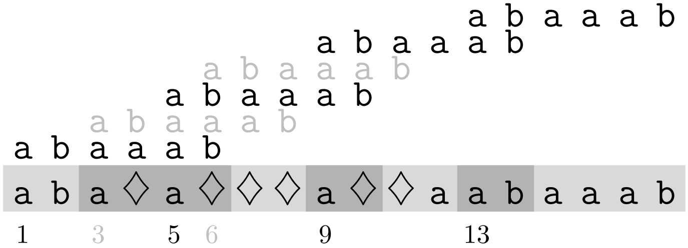

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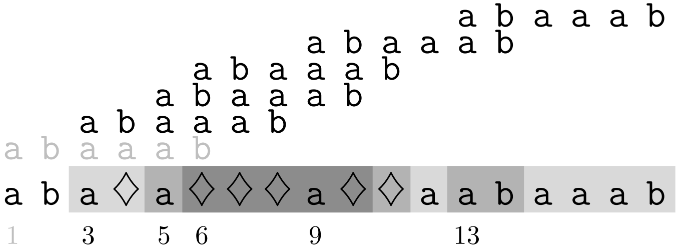
When $\mathcal{P}$ is a covering set?



Definition

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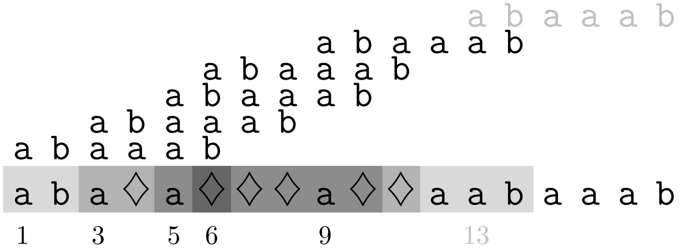
- $\min \mathcal{P} = 1$,

Covering set of a cover



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```
                              a b a a a b
                        a b a a a b
                  a b a a a b
                a b a a a b
            a b a a a b
        a b a a a b
        a b a ◇ a ◇ ◇ ◇ a ◇ ◇ a a b a a a b
        1   3   5 6     9       13
```

Definition

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When $\mathcal{P}$ is a covering set?

- $\min \mathcal{P} = 1$,
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- $|q - p| \leq |S|$ for any two consecutive $p, q \in \mathcal{P}$.

How to check covering sets?

- We could miss covers S with $Occ(S,T) \subseteq \mathcal{A}$.
- We will find covers which admit a covering set $\mathcal{P} \subseteq \mathcal{A}$.
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◊ a ◊ a b a ◊ a b a b a a ◊ a ◊ a b

i	ℓ	1	3	6	13	15
1						
3						
5						
7						
12						
14						
16						

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$\diamondsuit$ a $\diamondsuit$ a b a $\diamondsuit$ a b a b a a $\diamondsuit$ a $\diamondsuit$ a b

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i	ℓ	1	3	6	13	15
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3						
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♢ a ♢ a b a ♢ a b a b a a ♢ a ♢ a b

i	ℓ	1	3	6	13	15
1	**18**	♢	♢	♢	♢	♢
3	10	♢	b	b	-	-
5						
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◊ a ◊ a b a ◊ a b a b a a ◊ a ◊ a b

i	ℓ	1	3	6	13	15
1	**18**	◊	◊	◊	◊	◊
3	10	◊	b	b	-	-
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♢ a ♢ a b a ♢ a b a b a a ♢ a ♢ a b

i	ℓ	1	3	6	13	15
1	**18**	♢	♢	♢	♢	♢
3	10	♢	b	b	-	-
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i	ℓ	1	3	6	13	15
1	**18**	◇	◇	◇	◇	◇
3	10	◇	b	b	-	-
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1	**18**	◇	◇	◇	◇	◇
3	10	◇	b	b	-	-
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3	10	$\diamondsuit$	b	b	-	-
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$|S| = 3 \quad 12 - 7 > 3$ NO

i	ℓ	1	3	6	13	15
1	**18**	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$
3	10	$\diamondsuit$	b	b	-	-
5	8	b	$\diamondsuit$	b	-	-
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i	ℓ	1	3	6	13	15
1	**18**	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$
3	10	$\diamondsuit$	b	b	-	-
5	8	b	$\diamondsuit$	b	-	-
7	8	$\diamondsuit$	b	a	-	-
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$|S| = 7$ mismatch at position 6 NO

i	ℓ	1	3	6	13	15
1	**18**	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$
3	10	$\diamondsuit$	b	b	-	-
5	8	b	$\diamondsuit$	b	-	-
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i	ℓ	1	3	6	13	15
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3	10	$\diamondsuit$	b	b	-	-
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$|S| = 5$ $S[1] = \mathtt{a}$ $S[3] = \mathtt{b}$ YES

i	ℓ	1	3	6	13	15
1	**18**	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$
3	10	$\diamondsuit$	b	b	-	-
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7	8	$\diamondsuit$	b	a	-	-
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14	**5**	$\diamondsuit$	$\diamondsuit$	-	-	-
16	**3**	$\diamondsuit$	b	-	-	-

Do we need to verify all subsets?

We could only miss covers S such that for each $i \in Occ(S,T)$ we have $T[1+j] = T[i+j] = \diamondsuit$ for some $j < |S|$.

- Every occurrence of $i \in \mathcal{P}$ covers a non-solid position.

Do we need to verify all subsets?

We could only miss covers S such that for each $i \in Occ(S,T)$ we have $T[1+j] = T[i+j] = \Diamond$ for some $j < |S|$.

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```
                        a b a a a b
                a b a a a b
          a b a a a b
        a b a a a b
    a b a a a b
a b a a a b
a b a ◊ a ◊ ◊ ◊ a ◊ ◊ a a b a a a b
```

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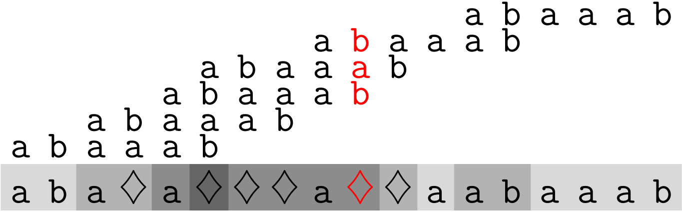

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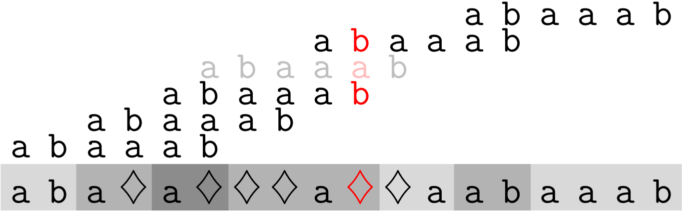

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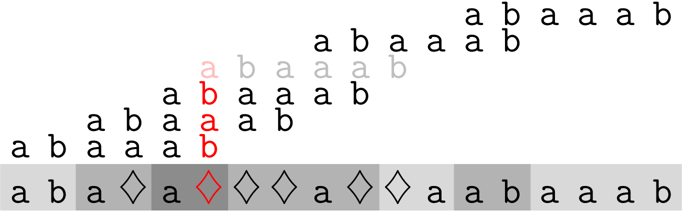

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```
                            a b a a a b
                        a b a a a b
                    a b a a a b
                a b a a a b
            a b a a a b
        a b a a a b
        a b a ◇ a ◇ ◇ ◇ a ◇ ◇ a a b a a a b
```

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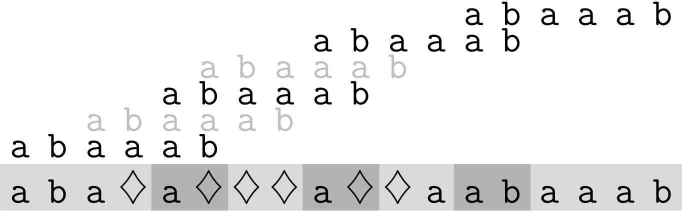

- If $\mathcal{P}$ is minimal, every position is covered by 1 or 2 occurrences $i \in \mathcal{P}$.

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```
                        a b a a a b
                a b a a a b
          a b a a a b
        a b a a a b
    a b a a a b
a b a a a b
a b a ◇ a ◇ ◇ ◇ a ◇ ◇ a a b a a a b
```

- If $\mathcal{P}$ is minimal, every position is covered by 1 or 2 occurrences $i \in \mathcal{P}$.
- It suffices to consider $\mathcal{P}$ with $|\mathcal{P}| \le 2k$,

$$\binom{|\mathcal{A}|}{\le 2k} = \mathcal{O}(2^{2k \log |A|}) = 2^{\mathcal{O}(k \log k)}.$$

How to fill few holes?

i	ℓ	1	3	6	13	15
1	**18**	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$
3	10	$\diamondsuit$	b	b	-	-
5	8	b	$\diamondsuit$	b	-	-
7	8	$\diamondsuit$	b	a	-	-
12	**7**	a	$\diamondsuit$	b	-	-
14	**5**	$\diamondsuit$	$\diamondsuit$	-	-	-
16	**3**	$\diamondsuit$	b	-	-	-

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14	**5**	$\diamondsuit$	$\diamondsuit$	-	-	-
16	**3**	$\diamondsuit$	b	-	-	-

Can guessing still help?

1. $\max \mathcal{P} = |T| - |S| + 1$ ($\mathcal{O}(|\mathcal{A}|)$ choices)

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1. $\max \mathcal{P} = |T| - |S| + 1$ ($\mathcal{O}(|\mathcal{A}|)$ choices)
2. Some $i \in \mathcal{P}$ ($\mathcal{O}(|\mathcal{A}|)$ choices).

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i	ℓ	1	3	6	13	15
1	**18**	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$
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5	8	b	$\diamondsuit$	b	-	-
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3. How to fill $\diamondsuit$'s in the i-th row $\quad (\mathcal{O}(|\mathcal{A}|^h)$ choices).
   Fail if more than h $\diamondsuit$'s, check candidates otherwise.

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i	ℓ	1	3	6	13	15
1	**18**	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$	$\diamondsuit$
3	10	$\diamondsuit$	b	b	-	-
5	8	b	$\diamondsuit$	b	-	-
7	8	b	b	a	-	-
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Which subsets do we still need to check?

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- $T[1..|S|] \cap T[i..i+|S|-1]$ has at least $h+1$ holes for each $i \in Occ(S,T)$.

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Which covers could we still miss?

- $T[1..|S|] \cap T[i..i+|S|-1]$ has at least $h+1$ holes for each $i \in Occ(S,T)$.
- Every occurrence $i \in \mathcal{P}$ covers at least $h+1$ non-solid symbols.
- $|P| \le 2\frac{k}{h+1}$ if $\mathcal{P}$ is minimal.

$$\binom{|\mathcal{A}|}{\le 2\frac{k}{h+1}} = 2^{\mathcal{O}\left(\frac{k \log k}{h}\right)}.$$

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In total:

- $\mathcal{O}(|\mathcal{A}|^{h+2}) = 2^{\mathcal{O}(h \log k)}$ for guessing,
- $\mathcal{O}(|\mathcal{A}|^{h+2}) = 2^{\mathcal{O}(\frac{k}{h} \log k)}$ for finding missed covers,
- $2^{\mathcal{O}(\sqrt{k} \log k)}$ for $h = \sqrt{k}$.

Conclusions and open problems

Our results:

- $\mathcal{O}(nk^4 + 2^{\mathcal{O}(k \log k)})$-time algorithm,
- $\mathcal{O}(nk^4 + 2^{\mathcal{O}(\sqrt{k} \log k)})$-time version for partial words,
- $2^{o(\sqrt{k})} n^{\mathcal{O}(1)}$ lower bound already for binary partial words (under ETH).

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Open problems:

- Are indeterminate strings inherently harder?
  - For constant alphabets, no!
- What is the right exponent for partial words?
  - For constant alphabets, at most $\mathcal{O}(\sqrt{k \log k})$!

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Our results:

- $\mathcal{O}(nk^4 + 2^{\mathcal{O}(k \log k)})$-time algorithm,
- $\mathcal{O}(nk^4 + 2^{\mathcal{O}(\sqrt{k} \log k)})$-time version for partial words,
- $2^{o(\sqrt{k})} n^{\mathcal{O}(1)}$ lower bound already for binary partial words (under ETH).

Open problems:

- Are indeterminate strings inherently harder?
  - For constant alphabets, no!
- What is the right exponent for partial words?
  - For constant alphabets, at most $\mathcal{O}(\sqrt{k \log k})$!
- Polynomial kernel?
  - For partial words, yes! (obtained through NP-hardness)

Thank you

Thank you for your attention!

Full version available at `http://arxiv.org/abs/1412.3696`.