Approximating Upper Degree-Constrained Partial Orientations

Marek Cygan and Tomasz Kociumaka

Institute of Informatics
University of Warsaw

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Problem Definition

Upper Degree-Constrained Partial Orientation (UDPO)

Input: undirected graph G,
degree constraints $d^+, d^- : V(G) \to \mathbb{Z}_{\geq 0}$.

Find: A subset $\overline{F} \subseteq E(G)$ and its orientation F such that:

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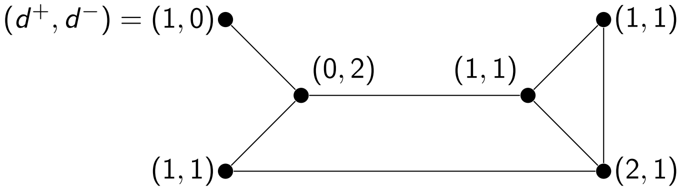

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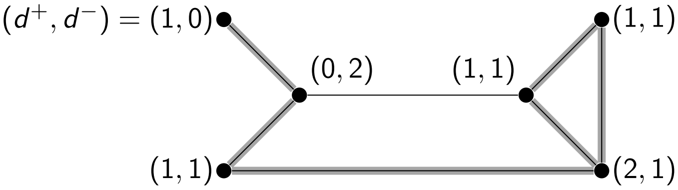

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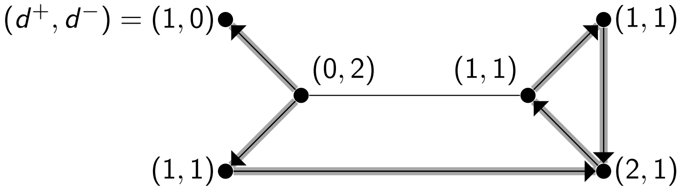

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Previous work: Gabow, SODA 2006

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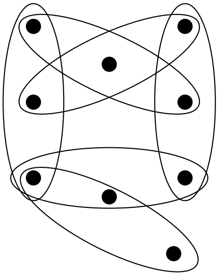

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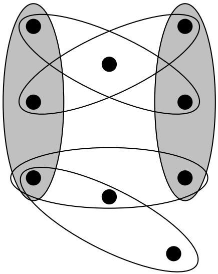

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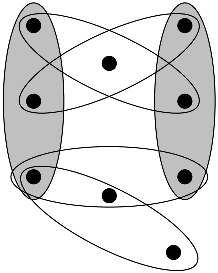
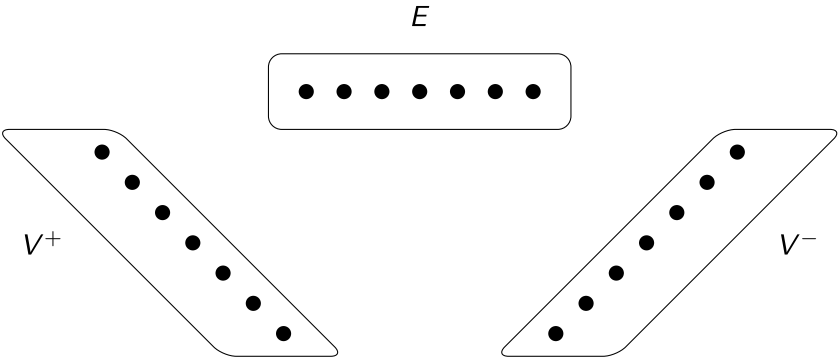


3-Dimensional Matching

Input: a universe $U = X \uplus Y \uplus Z$, a family $\mathcal{F} \subseteq X \times Y \times Z$.
Goal: find a maximum-size subfamily of $\mathcal{F}$ of pairwise disjoint sets.

- V^+ $d^+(v)$ copies of each vertex $v \in V$,
- V^- $d^-(v)$ copies of each vertex $v \in V$,
- E the set of (undirected) edges,
- $\mathcal{F}$ (v_i^+, u_j^-, e) and (u_i^+, v_j^-, e) for each $e = uv$.



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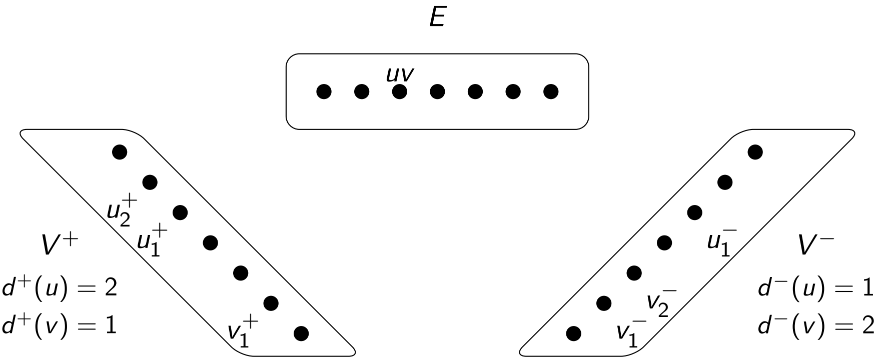

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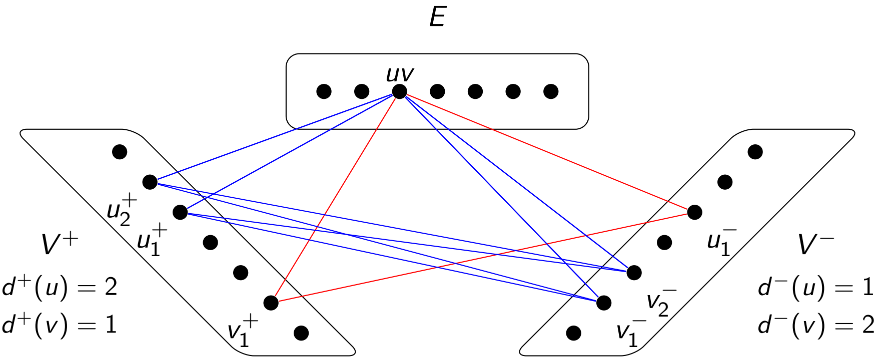

p-local search for k-Set Packing

Set $\mathcal{A} = \emptyset$.
While there exists $Y \subseteq \mathcal{F}$ such that:

- the symmetric difference $\mathcal{A}\Delta Y$ consists of disjoint sets,
- $|\mathcal{A}\Delta Y| > |\mathcal{A}|$,
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We then call $Y \setminus \mathcal{A}$ a p-improving set.

Approximation Ratios

Approximation ratios of p-local search for k-Set Packing and UPDO (upper bounds).

author	p	k-SP	UDPO
folklore	1	k	
folklore	2	$\frac{1}{2}(k+1)$	
Hurkens & Schrijver [1989]	$\mathcal{O}(1)$	$\frac{1}{2}(k+\varepsilon)$	
Cygan et al. [2013]	$\mathcal{O}(\log n)$	$\frac{1}{3}(k+1+\varepsilon)$	
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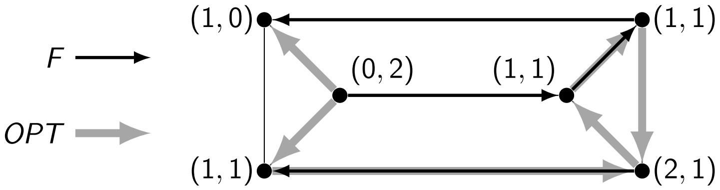

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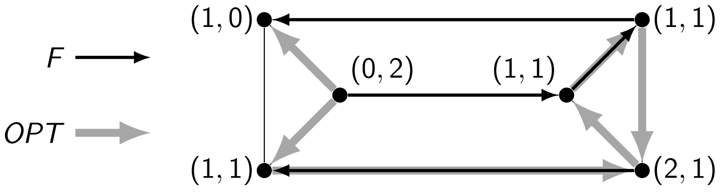

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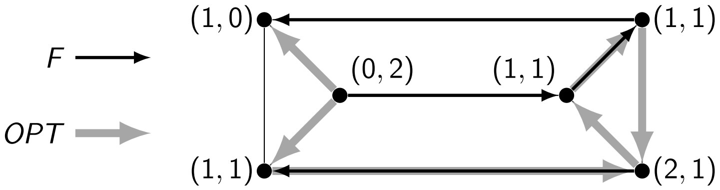

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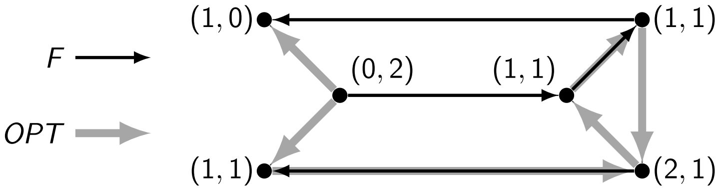

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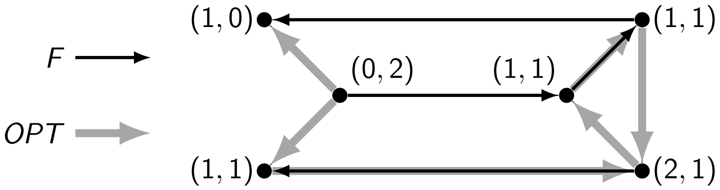

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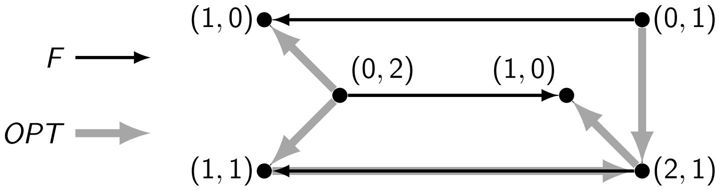

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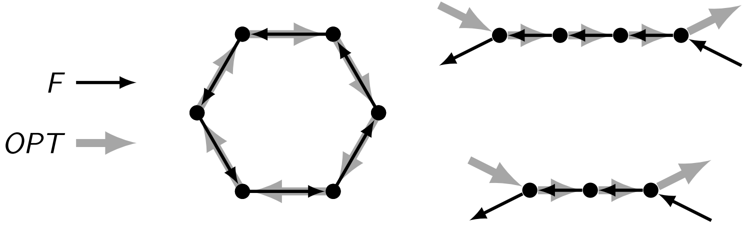
- any feasible solution in a set of paths and cycles.



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For an arbitrary instance I, and two local optima A and B, there is a simple instance I' with local optima A' and B' satisfying $|A| = |A'|$ and $|B| = |B'|$.

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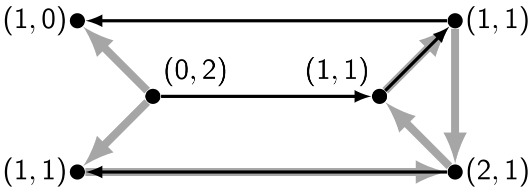

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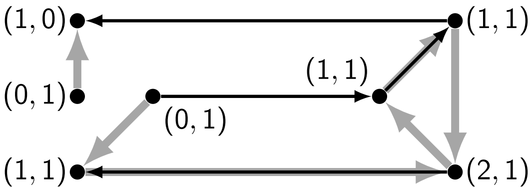

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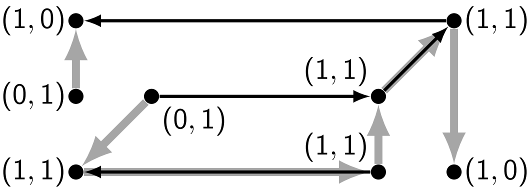

Few Common Edges: Improved Analysis

Conflicts between edges in F and OPT are represented in a graph.

- bipartite,
- degrees between 1 and 3.

Lemma

$3|OPT| \le \sout{4(1+\varepsilon)|F|}\ 3(1+\varepsilon)|F| + |\{e \in F : \deg(e) = 3\}|$

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Corollary

If $|\overline{F} \cap \overline{OPT}| \leq \frac{3}{4}|F|$, then F is a $(5/4+\varepsilon)$-approximate solution.

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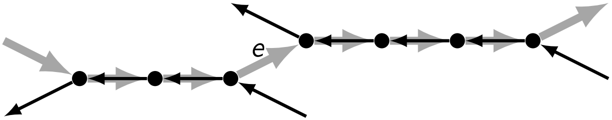

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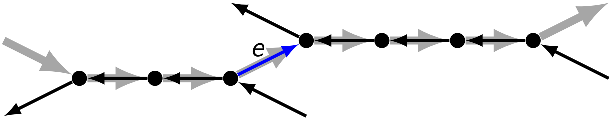

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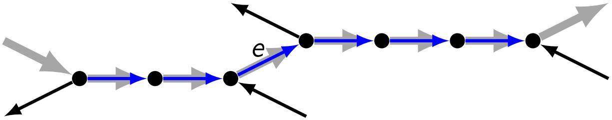

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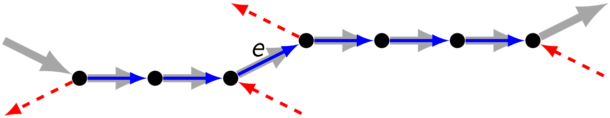

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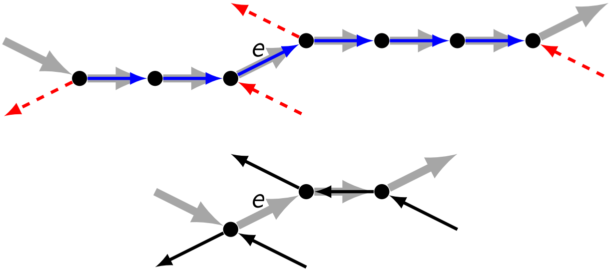

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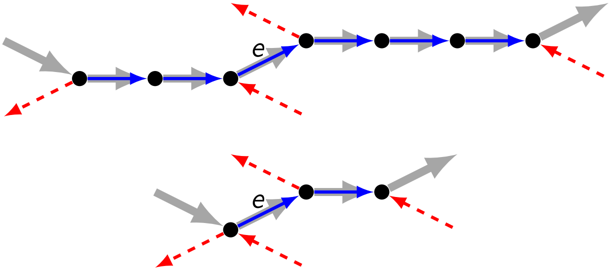

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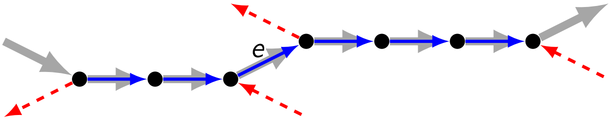
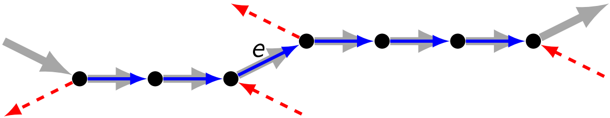


Build a conflict graph between $\overline{OPT} \setminus \overline{F}$ and $\overline{F} \setminus \overline{OPT}$.



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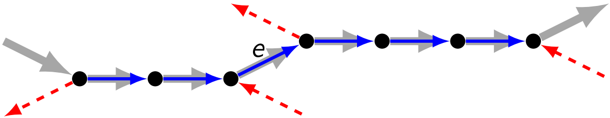

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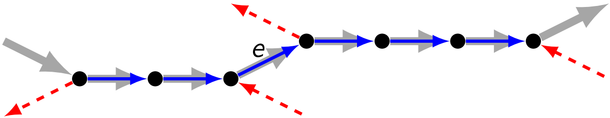

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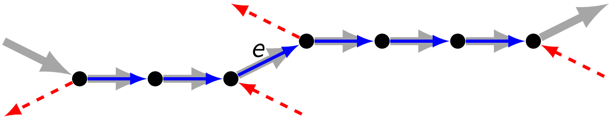
Many Common Edges: 4-Set Packing



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Theorem

Local search maximum F satisfies $|\overline{OPT} \setminus \overline{F}| \leq 2|\overline{F} \setminus \overline{OPT}| + \varepsilon|F|$.
$|OPT| \leq (1+\varepsilon)|F| + |\overline{F} \setminus \overline{OPT}| \leq (\frac{5}{4}+\varepsilon)|F|$ *if* $|\overline{F} \setminus \overline{OPT}| \leq \frac{1}{4}|F|$.

Conclusions and Open Problems

Our results:

1. Local-search algorithms for 3-SET PACKING perform better on UDPO instances:
   - $\mathcal{O}(1)$-local-search (Hurkens & Schrijver, 1989): $4/3 + \varepsilon$,
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- Tight example? Improved analysis?
  - One can restrict to simple instances.
- Improved approximation ratio?
  - $\mathcal{O}(\log n)$-local search: $(11/9 + \varepsilon)$! (not in the paper),
  - quasipolynomial running time,
  - polynomial-time $(11/9 + \varepsilon)$-approximation?

Questions?

Thank you for your attention!

INNOVATIVE ECONOMY
NATIONAL COHESION STRATEGY

FNP
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EUROPEAN REGIONAL DEVELOPMENT FUND

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