MATHEMATICAL MODEL OF THE TURBULENT FLOW BY MAREK BURNAT

In the simplest case the equations of the model are as follows

\[ \frac{\partial \rho}{\partial t}(t, \vec{x}, \alpha) + \alpha \nabla_x \rho(t, \vec{x}, \alpha) - \nu \Delta_x \rho(t, \vec{x}, \alpha) + -\kappa \int_{\mathcal{A}} M(t, \vec{x}, \alpha, \beta, \rho(t, \vec{x}, \alpha), \rho(t, \vec{x}, \beta)) d\beta = 0. \]

Clearly the above equations have to be completed by certain convenient initial and boundary conditions. The function to be found is \( \rho(t, \vec{x}, \alpha) \), it is the \( \alpha \)-mass density function. So called 'Euler mass density function' can be computed by integration

\[ \rho(t, \vec{x}) = \int_{\mathcal{A}} \rho(t, \vec{x}, \alpha) d\alpha. \]

The independent variables of the model are

t- time, \( \vec{x} \in \Omega \subset \mathbb{R}^3 \) - space variables, \( \alpha \in \mathcal{A} \subset \mathbb{R}^3 \) - "\( \alpha \)-velocities". The nonlinear function \( M \) of several variables is called the mixer. It serves to mix together various elements of the set

\[ \{ \rho(t, \vec{x}, \alpha) \mid \vec{x} \in \Omega, \quad \alpha \in \mathcal{A} \} \]

in such a way that the most important conservation laws are satisfied. Some details of this model and of its first one-dimensional numerical approximation can be found in [1].

Now, from the beginning of 2010 the new 2-dimensional approximation of the model have been implemented on the cluster halo2 (ICM U.W.). It is worth to note that in two-dimensional model the sets \( \Omega \) and \( \mathcal{A} \) are of the dimension 2. This means that the problem (if we don’t take into consideration the time variable \( t \)) is of the dimension 4.

References