Induced packings of cycles

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Cycle / packing problems

Old and well-studied:

- stable set;
- Hamiltonian cycle;
- Eulerian circuits;
- Erdős-Pósa problems;
- maximum cycle packing;
- cycle double cover conjecture;
- . . .

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In this talk: *Does G contain k cycles?*

What kind of packing?

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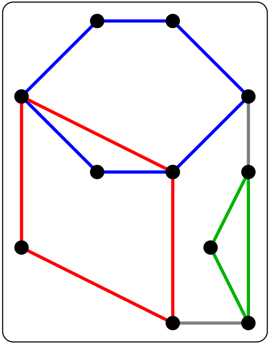

Edge-disjoint

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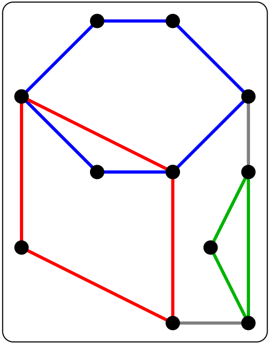

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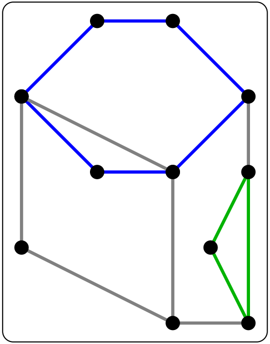

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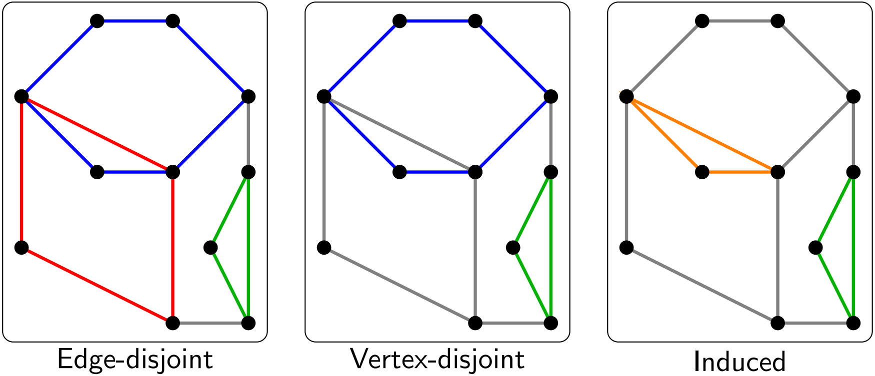

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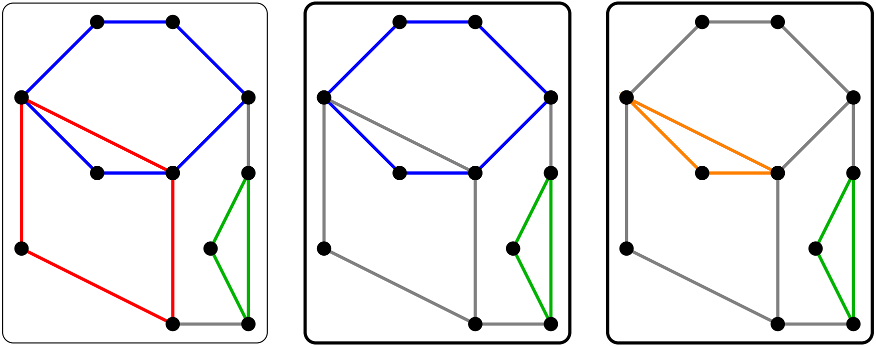

Edge-disjoint Vertex-disjoint Induced

Interesting?

Extends stable set and matching:

stable set: induced packing of K_1's;

matching: packing of K_2's (+ induced);

cycle pkg: packing of K_3-models (+ induced).

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Complexity questions:

- compare induced / non-induced versions of the same problem;
- induced minor testing complexity.

Parameterized complexity in a nutshell

- Parameterized problem:
- FPT:
- $f(k)$-kernel:
- $W[1]$-hard:

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- $W[1]$-hard: problems that are not FPT, e.g. CLIQUE (under widely-believed complexity-theoretic assumptions).

Parameterized problems for cycle packing

Disjoint Cycles

Input: (G, k)

Parameter: k

Question: Does G have k vertex-disjoint cycles?

Parameterized problems for cycle packing

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INDUCED CYCLES

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Question: Does G have k induced cycles?

Parameterized problems for cycle packing

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Input: (G, k)

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Question: Does G have k vertex-disjoint cycles?

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Both **NP-hard** (if we forget the parameter).

Complexity of cycle packing

Disjoint Cycles:

- FPT (minor testing) [GM];

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INDUCED CYCLES:

?

1st reduction: from Disjoint to Induced

$\boxed{(G, k) \text{ inst. of Disj. Cycles}} \rightarrow \boxed{(H, k) \text{ inst. of Ind. Cycles}}$

H: subdivide every edge of G

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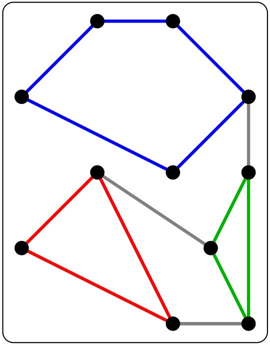
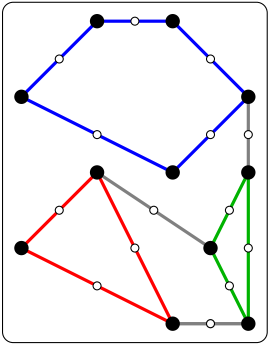
Equivalent instances?



G



H

1st reduction: from Disjoint to Induced

(G, k) inst. of Disj. Cycles $\rightarrow$ (H, k) inst. of Ind. Cycles

H: subdivide every edge of G

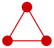
Consequence:

- $f(k)$-kernel for Ind. Cycles $\rightsquigarrow$ $f(k)$-kernel for Disj. Cycles;
- $\Rightarrow$ **no** polynomial kernel for Induced Cycles

(unless $\mathrm{NP} \subseteq \mathrm{coNP/poly}$).

2^{nd} reduction: from Stable Set to Induced Cycles

(G, k) inst. of Stable Set $\rightarrow$ (H, k) inst. of Ind. Cycles

H: glue a  to every vertex of G

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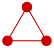
H: glue a  to every vertex of G

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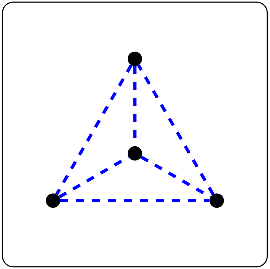

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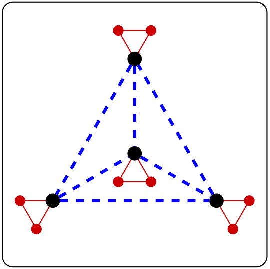
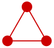


H

2^{nd} reduction: from STABLE SET to INDUCED CYCLES

(G, k) inst. of STABLE SET $\rightarrow$ (H, k) inst. of IND. CYCLES

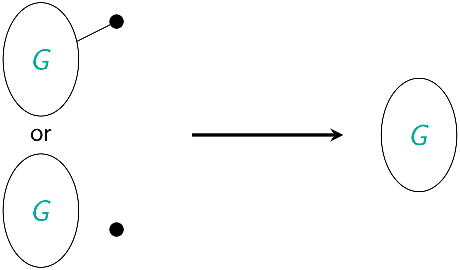
H: glue a  to every vertex of G

Consequence:

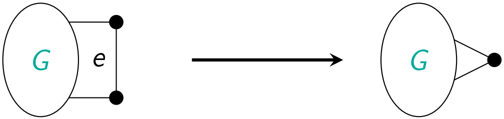
- FPT algo for IND. CYCLES $\rightsquigarrow$ FPT algo for STABLE SET;
- $\Rightarrow$ INDUCED CYCLES is $W[1]$**-hard**.

Two simple rules

(R1)



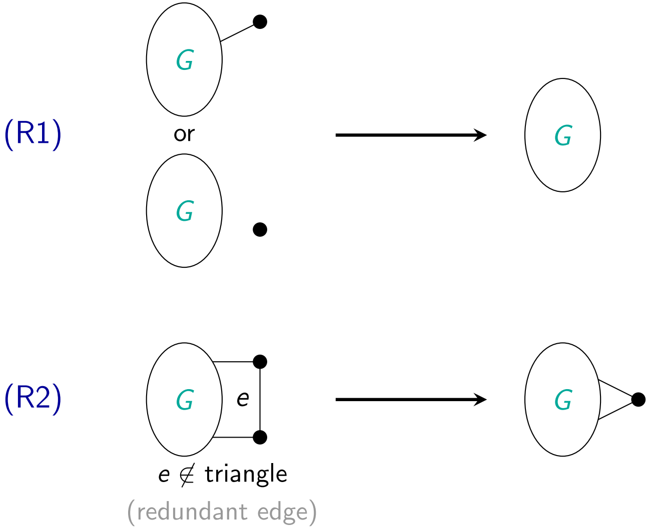
(R2)



$e \notin$ triangle

(redundant edge)

Two simple rules



Can be performed in linear time.

Attack plan

Goal: show that *large* reduced graphs contain k induced cycles.

(large $= f(\Delta, k)$)

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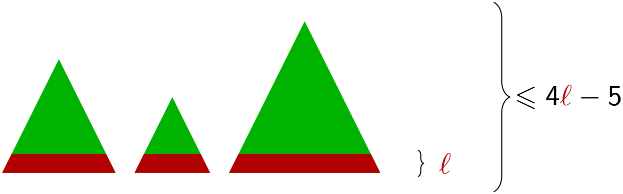
Very rough idea: by induction on k.

Forest without redundant edges

1. F forest, $\ell \geqslant 2$ leaves and no redundant edge $\Rightarrow |\mathrm{V}(F)| \leqslant 4\ell - 5$.



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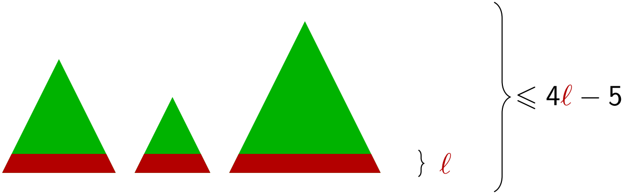

Proof with tree T and T' obtained by dissolution of degree-two vertices:

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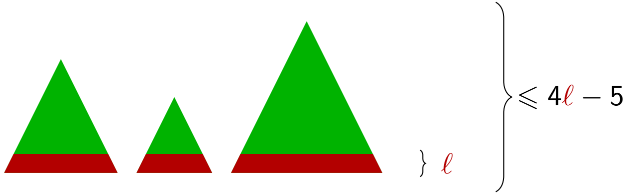

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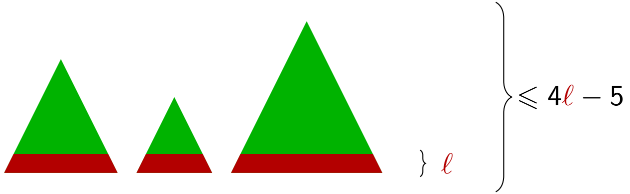

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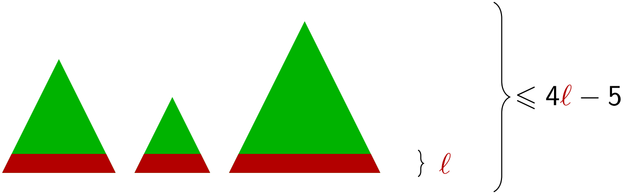

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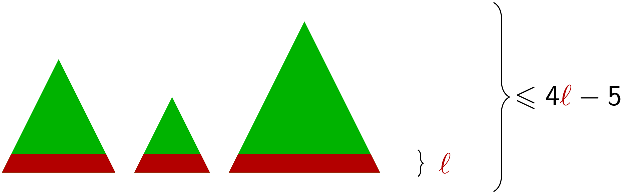

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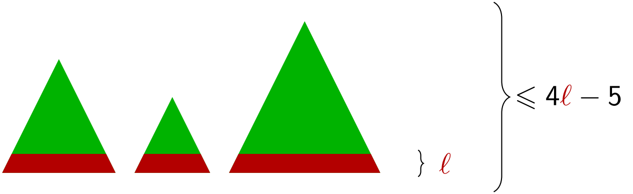

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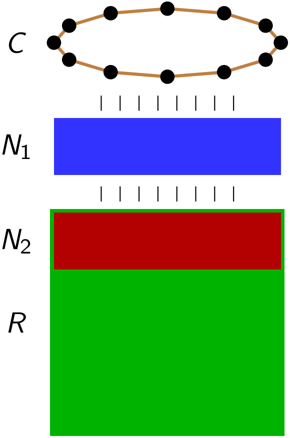
$$4\ell - 5 \geqslant |\mathrm{V}(T)|$$

Reduced graphs without two ind. cycles have small girth



Assuming $|C| = g > 10$:

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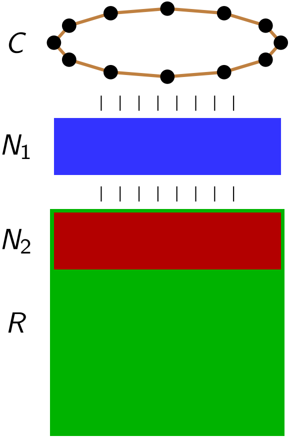

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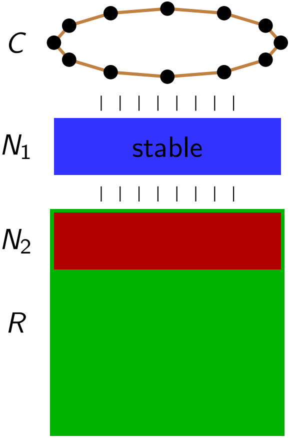

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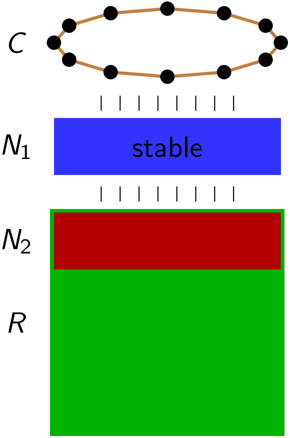

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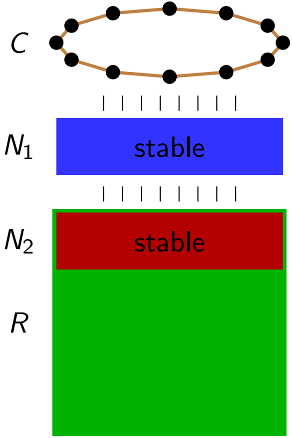

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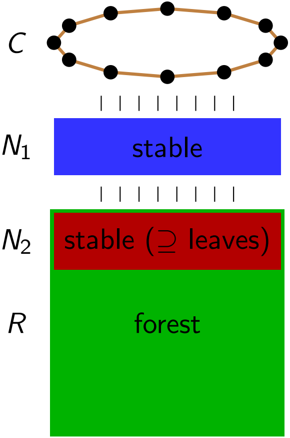

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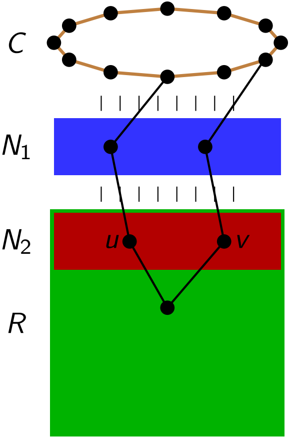

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f) $u, v \in N_2$ at distance 2 in R belong to a short cycle.

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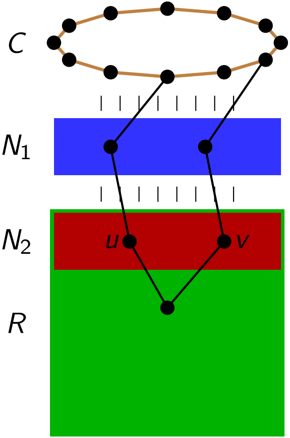

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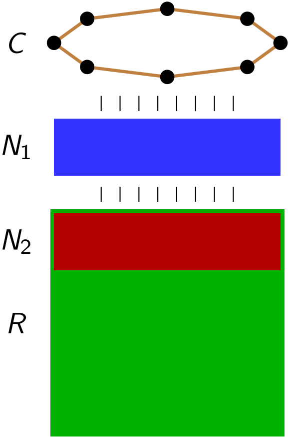
2. G reduced without 2 induced cycles $\Rightarrow g \leqslant 10$

Reduced graphs without two induced cycles are small



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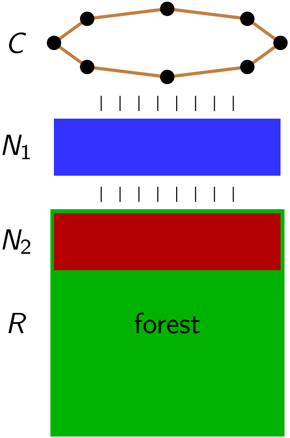

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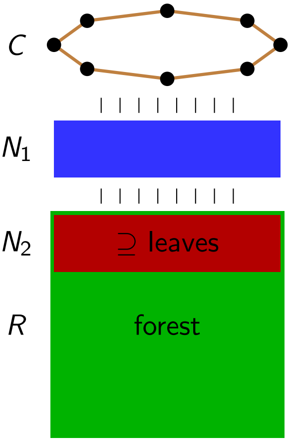

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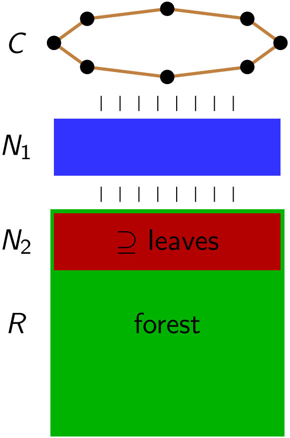

$|C| = g$

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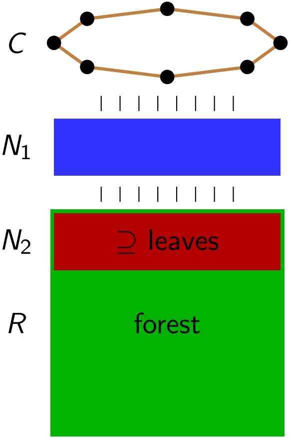

$|C| = g$

$|N_1| \leqslant g(\Delta - 2)$

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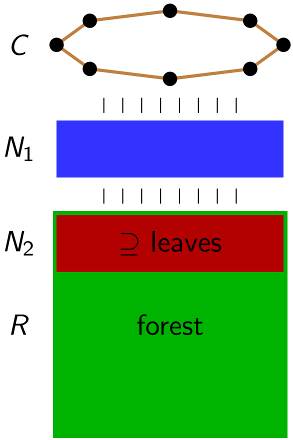

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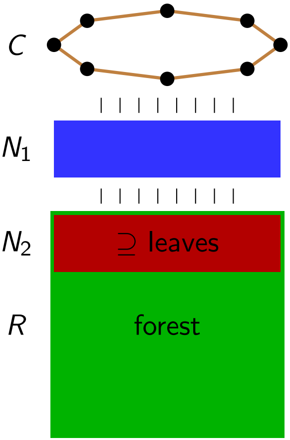

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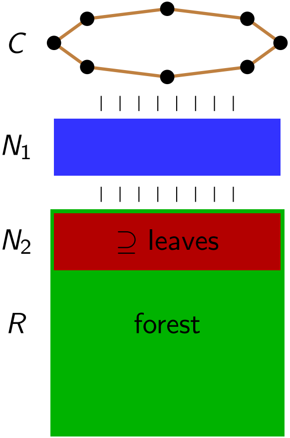
$|N_2| \leqslant g(\Delta - 2)(\Delta - 1)$

$|R| \leqslant 4g(\Delta - 2)(\Delta - 1) - 5$



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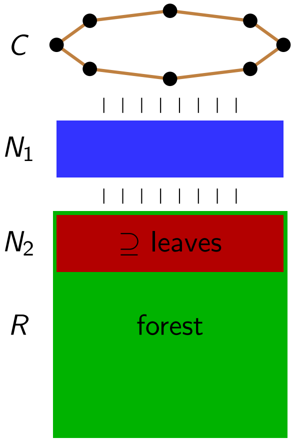
$$|G| \leqslant 4g\Delta^2$$

Reduced graphs without two induced cycles are small



$|C| = g$

$|N_1| \leqslant g(\Delta - 2)$

$+ \quad |R| \leqslant 4g(\Delta - 2)(\Delta - 1) - 5$

$|G| \leqslant 4g\Delta^2$

3. G reduced without 2 induced cycles $\Rightarrow |G| \leqslant 40\Delta^2$

Refining the attack plan

We just proved: G reduced and $|G| > 40\Delta^2 \Rightarrow G$ has two induced cycles.

(base case)

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Current goal: estimate the least $h_k : \mathbb{N} \to \mathbb{N}$ such that $\forall G$ reduced,

$$|G| > h_k(\Delta) \Rightarrow G \text{ has } k \text{ induced cycles.}$$

Refining the attack plan

We just proved: G reduced and $|G| > 40\Delta^2 \Rightarrow G$ has two induced cycles.
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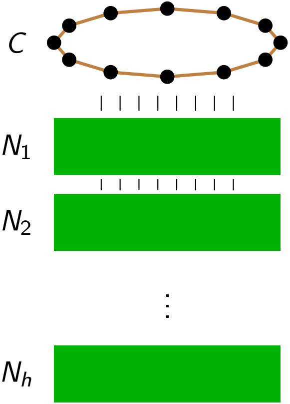
What we know: $h_2(d) \leqslant 40d^2$.

Reduced graphs of large girth have many vertices



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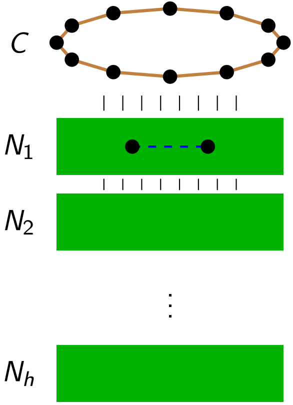

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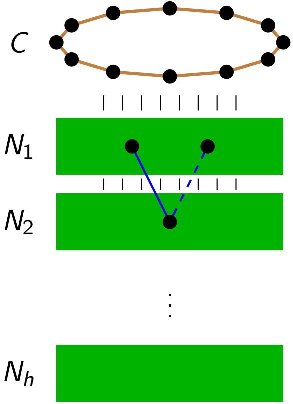

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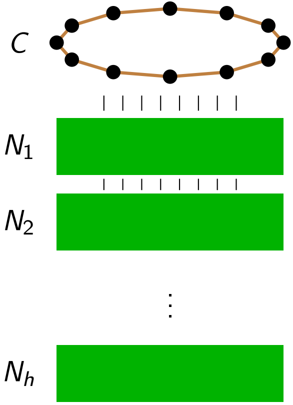

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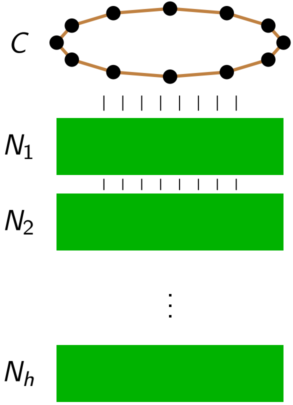

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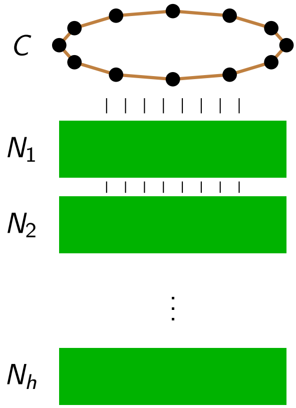

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e) $|G| \geqslant |C| + \sum_i N_i \geqslant g \cdot 2^{\lfloor g/8 \rfloor - 1}$.

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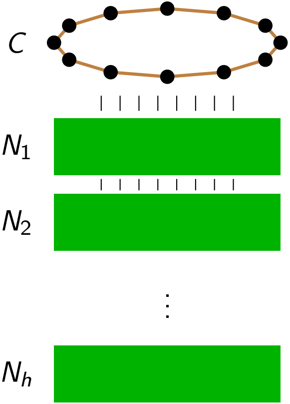

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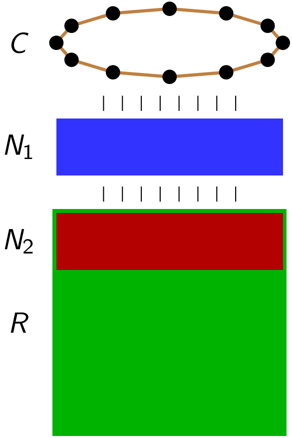
4. G reduced and $g \geqslant 6 \Rightarrow |G| \geqslant g \cdot 2^{\lfloor g/8 \rfloor - 1}$.

Reduced graphs not having many induced cycles are small



Assuming that $G \not\supseteq k$ induced cycles:

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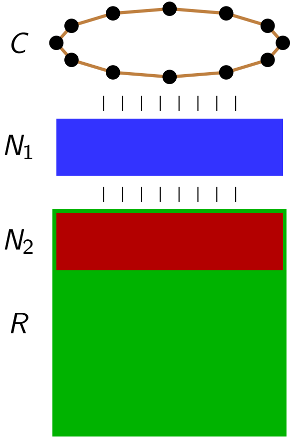

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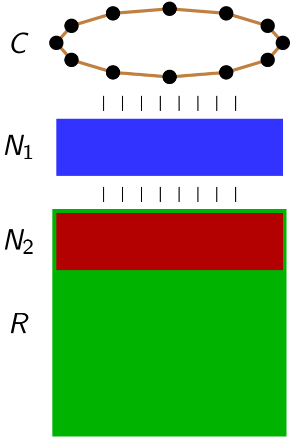

Assuming that $G \not\supseteq k$ induced cycles:

a) $R \not\supseteq (k-1)$ induced cycles

b) $|R| \leqslant h_{k-1}(\Delta)$;

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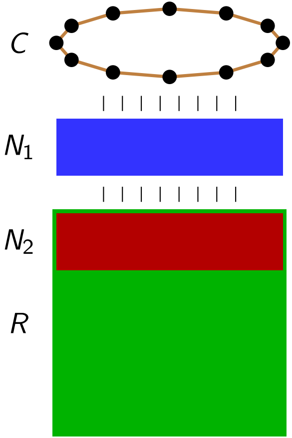

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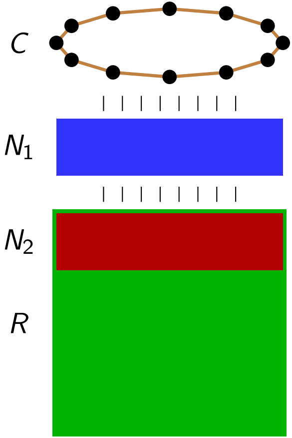

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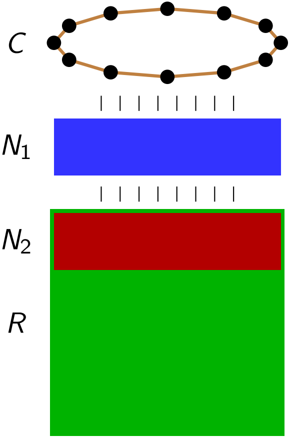

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5. G reduced without $k \geqslant 2$ induced cycles $\Rightarrow |G| \leqslant 4g\Delta^2 + h_{k-1}(\Delta)$.

The crash

$$g \cdot 2^{\lfloor g/8 \rfloor - 1} \overset{\textbf{4.}}{\leqslant} |G|$$

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Remarks:

- two parameters: k and Δ;
- explicit constants;
- runs in linear time;
- only check the size (still a kernel?).

Can we improve this kernel?

Kernel obtained by upper bounding the least $h_k : \mathbb{N} \to \mathbb{N}$ s.t. $\forall G$ reduced,

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$\Rightarrow$ no $o(k\Delta^2)$-kernel with this technique.

Connections with DISJOINT CYCLES

Reminder (on DISJOINT CYCLES)**:**

- poly reduction from DISJOINT CYCLES to INDUCED CYCLES;
- $O(k \log k)$-kernel in graphs with no induced $K_{1,t}$ [Fomin et al. 2011]
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Consequence: $O(\Delta^2 k \log \Delta k)$-kernel for DISJOINT CYCLES.

Connections with Disjoint Cycles

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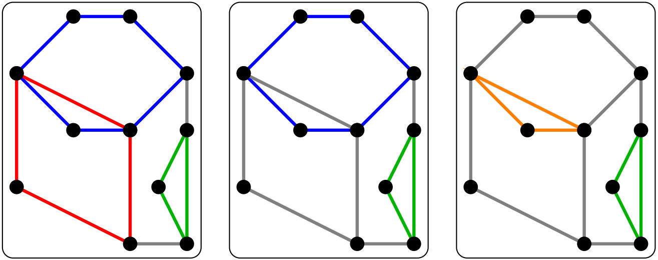
  $\rightsquigarrow O(\Delta k \log k)$-kernel in general.

Consequence: $O(\Delta^2 k \log \Delta k)$-kernel for Disjoint Cycles.

Worse than the kernel of [Fomin et al. 2011], however:

- simple proof (no protrusions!);
- explicit (small) constants.

Extensions?



Edge-disjoint $d \geqslant 0$ | Vertex-disjoint $d \geqslant 1$ | Induced $d \geqslant 2$

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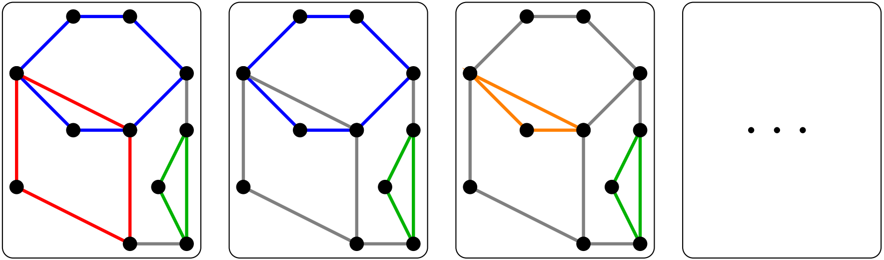

Edge-disjoint — $d \geqslant 0$

Vertex-disjoint — $d \geqslant 1$

Induced — $d \geqslant 2$

$\cdots$ — $d > 2$

Recap

- $O(\Delta^2 k \log \Delta k)$-kernel for Disjoint Cycles and Induced Cycles (when parameterized by k and Δ);
- simple proof, 2 reduction rules;
- explicit constants;
- runs in linear time;
- almost best possible with this technique.

Further research

- extensions to *scattered packings* of cycles?

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- $O(\Delta^2 k)$-kernel for INDUCED CYCLES with this technique?
- better kernel for DISJOINT CYCLES?

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- $O(\Delta^2 k)$-kernel for Induced Cycles with this technique?
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Thank you!