Singular collective dynamics

Jan Peszek

Research project objectives. Collective dynamics (or dynamics of interacting particles) originates from the mathematical description of the motion of particles that interact through a potential field. Recently, models with such a structure become increasingly popular, partially due to a significant development of technology. These are the models of self-propelled agents with certain predefined tendencies, such as a tendency to aggregate, disperse, align or otherwise synchronize into a particular pattern. Examples of described phenomena include flocking of birds and schooling of fish, but also, seemingly unrelated, consensus among individuals, distribution of goods, emergence of languages and synchronization of unmanned vehicles. Collective dynamics is a key-word that refers to such models.

The aim of the project is to perform a multiscale (from *microscopic* through *mesoscopic* to *macroscopic*) mathematical analysis of models of collective dynamics. The project is focused on models with singular interactions and on the propagation of information for models with compactly supported range of interactions/communication. The objectives are as follows.

- **(A): Limiting passage from micro- to mesoscopic description for weakly and strongly singular Cucker-Smale interactions.**
- **(B): Topological approach to propagation of information with compactly supported range of communication.**
- **(C): Analysis of large-crowd alignment dynamics in measure-dependent Besov spaces.**

Research project methodology. Methodology includes the standard toolbox of ODEs and PDEs, such as fixed point theory, energy methods, continuity method, particle methods, iterative decoupling schemes, Galerkin approximation, Sobolev-Slobodecki spaces, Campanato spaces, renormalized solutions in the spirit of DiPerna-Lions, Paley-Littlewood decomposition for Besov spaces, BBGKY hierarchy, Fourier transform, Riesz potentials, Calderon-Zygmund theory, K-method of interpolation and many other methods. The following techniques are expected to be particularly useful: measure-dependent Besov spaces (developed for example in works by Hajłasz and Koskela), mean-field limit, optimal transport theory, Filipov's theory of solutions to singular ODEs, meshfree discretizations and Beale-Kato-Majda-type criteria.

Expected impact of the research project on the development of science. One of the fundamental challenges of mathematical physics, related to the famous Hilbert's 6th problem, is the rigorous derivation of the laws of the motion of continua from simple models of interacting particles. It connects three branches of differential equations: ODE systems (e.g. Newtonian systems), kinetic equations (e.g. Boltzmann and Vlasov equations) and hydrodynamical, large crowd systems (e.g. Euler system). The aim of the project is directly connected to this fundamental challenge, as it aims to perform such a rigorous *micro-meso-macroscopic* derivation in the case of models of collective dynamics.

Each of the project's objectives addresses a significant problem within the core mainstream of the collective dynamics. Moreover, the techniques and strategies applied within the project are expected to contribute also to the classical models of fluids and gasses related to Hilbert's 6th problem. Objective (A) focuses on the characterization of the development of Dirac masses in the *mesoscopic* scale as well as on the improvement of earlier results on the limiting passage in the weakly singular case. In the strongly singular case we aim to discretize arguments from the work by Do *et. al.*, *ARMA* 2018, to obtain the first result on the limiting passage from *micro-* to *mesoscopic* collective dynamics. Objective (B) is dedicated to a new and more elegant description of the propagation of information for models with compactly supported range of communication, using topological interactions in the spirit of a recent work by Shvydkoy and Tadmor. Objective (C) aims to provide the first results in the case of multidimensional large-crowd alignment dynamics and the first well-posedness results admitting the possibility of vacuum (which is interesting also from the point of view of classical fluid mechanics).