Singular collective dynamics

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What do flocks of birds, distribution of goods and gang related crime have in common? These seemingly distinct phenomena are a characteristic example of the fascinating flexibility of mathematics as a tool to describe the surrounding world. It turns out that the above phenomena are described by a single class of models known as **the models of collective dynamics**. Such models describe phenomena, in which a group of individuals (referred to as "particles") interact with each other non-locally, for example by visual or verbal communication. Interestingly enough, from the mathematical point of view, such interactions are very similar to another non-local interaction, witnessed by everybody on the daily basis – **the gravitational interaction**.

The goal of the project is the **multiscale** analysis of models of collective dynamics in two, particularly difficult cases. The first one, when the intensity of the interactions between the neighboring individuals can grow arbitrarily large (**singular interactions**); and the second one, when the **interactions have finite range** (which is very reasonable from a real-life, practical point of view, but is difficult mathematically).

Analysis within the project is **multiscale**, which can be compared to observing certain phenomena **under a microscope**, **with the naked eye** and **from a bird's eye view**. The closest analogy comes from observation of water. **Under a microscope**, water is a mixture of numerous tiny particles, which, like billiard balls, constantly and chaotically bounce off of each other. **With a naked eye** we do not perceive individual particles, but we observe the motion of the fluid itself, which flows unpredictably creating waves, whirlpools and streams. On the other hand, while looking, for example, at the Atlantic Ocean **from afar**, we cease to see the smaller waves and turbulences, but we begin noticing the mighty ocean currents and other large phenomena. Mathematics provides a way to describe how the chaotic motion of tiny particles of water turns into waves and streams and finally into powerful currents but it is a very difficult task. Another example of the shift of the scale in real-life phenomena comes from **economics**, where each individual decision translates into interactions between businesses and corporations which then affect the international market.

This research direction is justified both from the point of view of classical mathematical challenges related to the aforementioned multiscale analysis of water, but also from the perspective of applications of models of collective dynamics. The possibilities of applications are very wide: from flocks of birds to schools of fish and algal blooms. From emergence of languages in primitive societies to distribution of goods and to control over sensor networks and unmanned aerial vehicles. Recently, particularly impactful applications can be found in machine learning and the programming of AI, where the AI's decision-making process can be viewed as competition and cooperation of particles representing certain responses. The significance of the research on the collective dynamics increases with the development of new technologies, and we can safely say that currently we are only scratching the surface of its true potential.