Special assignment

Chose **one** of the following problems. Use of Matlab and other technology highly encouraged. The problems are stated in a slightly ambiguous way on purpose. Your job is to use all the knowledge from the course (outside knowledge is permitted but not required) and your creativity to deal with them. Grades will be assigned for accuracy (40%), effort (30%) and aesthetic and clear presentation and explanation (30%). Remember that this assignment is not obligatory.

Problem 1. A set of data is represented by the points on a plane:

$$A = \Big\{(-10,21),(-9,12),(-8,48),(-7,79),(-6,63),(-5,51),(-4,17),(-3,15),(-2,19),$$
$$(-1,11),(0,19),(1,-1),(2,48),(3,1),(4,-4),(5,13),(6,12),(7,40),(8,2),(9,73),(10,124)\Big\}.$$

The goal is as follows:

1. Find two curves, such that all the points in A are between the curves. Provide the exact formula for the curves.
2. The vertical distance between the curves is small (so that the data really "fits" between the curves).
3. The curves represent the data well, that is they do not oscillate more than the data itself.
4. You can disregard up to 3 points in A, due to the possible errors in measurement.
5. Provide a picture (I suggest pictures produced using technology instead of hand drawn).
6. Provide any information to convince the reader that your choice of curves is good. For example, if you decide to use the least-squares, provide the basis you used, the error (in this case it would be the vertical distance between the curves), compare it to different templates of least-squares curve fitting. In general your job is to convince the reader that the solution you provide is good and well thought.

Problem 2. Suppose that there are 5 states $S1, S2, S3, S4$ and $S5$. Suppose further that the citizens of each of the states travel according to the following **ticket rule**:

1. In each state there are exactly 12 tickets that induce the migration between the states.
2. Each state assigns the tickets to itself and to all other states. For example, S1 may assign 3 tickets to itself, 5 tickets to S2, 3 tickets to S3 and 1 ticket to S5 (thus it assigns 0 tickets to S4). Note that the total number of the assigned tickets is indeed 12.
3. The population of each states moves in proportions of assigned tickets. Thus in case of S1 above: $\frac{3}{12}$ of the population stays in S1, $\frac{5}{12}$ of the population moves to S2, $\frac{3}{12}$ of the population moves to S3, nobody moves to S4, and $\frac{1}{12}$ of the population moves to $S5$. Notice that with the ticket assignment for all the states, we obtain full information on the migration and we can produce the transition matrix.

Suppose that

- S1 assigns 3 tickets to itself, 5 tickets to S2, 3 tickets to S3 and 1 ticket to S5;

- S2 assigns 7 tickets to itself and 5 tickets to S1;
- S3 assigns 8 tickets to itself, 2 tickets to S2 and 2 tickets to S5;
- S4 assigns 4 tickets to itself, 2 tickets to S1, 2 tickets to S2, 2 tickets to S3 and 2 tickets to S5.

Your task is:

1. Assign the tickets of S5 in such a way that the steady state exists and is always achievable as a limit of iterations of the transition matrix.
2. Moreover, the steady state (let us denote it by v) has the lowest possible variance between its entries. The variance of its entries is defined as

$$S = (v_1 - m)^2 + (v_2 - m)^2 + (v_3 - m)^2 + (v_4 - m)^2 + (v_5 - m)^2, \qquad \text{where } m = \frac{v_1 + v_2 + v_3 + v_4 + v_5}{5}.$$

3. Explain your reasoning. If you cannot provide a rigorous mathematical proof that your solution minimizes the variance of the steady state, provide reasonable numerical evidence. As in Problem 1, your job is to convince the reader that you know what you are doing and your solution is good.

Problem 3. *Problem 3 automatically gets 0 points for aesthetics and clarity of presentation. This means that it can get you at most 70% of the maximum points. It is a lifeline.*

Read and understand Chapter 12 of the book. Solve Exercises 12.1, 12.3, 12.6 and 12.9.