In preparation for the final exam

- The final will take place during the class on Wednesday 12/05/2018 at 10:00 am and during the class on Friday 12/07/2018 at 10:00 am
- On Wednesday there will be 10-12 multiple choice questions out of the presented below 25 questions. Each of the questions is **multiple choice**, which means that, effectively, each question is a set of five questions with a yes/no answer. In particular, if you find one correct answer to any of the questions, it doesn't mean that the remaining answers are incorrect. I suggest going through these questions very carefully. During the exam the questions may be slightly changed (so don't just memorize the answers).
- On Friday there will be a collection of problems from midterms from 3 topics: **least-squares (including curve fitting)**, **markov chains (without Google Pagerank)** and **SVD**.
- I will do my very best to upload propositions of the grades on Friday.

Problem 1. Suppose that we have 3 sectors producing products P1 P2 and P3, respectively:

- To produce 1 unit of P1 it takes 0.1 unit of P1, 0.2 unit of P2 and 0.25 unit of P3,
- To produce 1 unit of P2 it takes 0.15 unit of P1 and 0.4 unit of P2,
- To produce 1 unit of P3 it takes 0.1 unit of P1, 0.3 unit of P2 and 0.2 unit of P3,

The internal demand matrix is equal to

$$A = \begin{bmatrix} 0.1 & 0.2 & 0.25 \\ 0.15 & 0.4 & 0 \\ 0.1 & 0.3 & 0.2 \end{bmatrix}^{-1} \qquad B = \begin{bmatrix} 0.1 & 0.2 & 0.25 \\ 0.15 & 0.4 & 0 \\ 0.1 & 0.3 & 0.2 \end{bmatrix} \qquad C = \begin{bmatrix} 0.1 & 0.2 & 0.25 \\ 0.15 & 0.4 & 0 \\ 0.1 & 0.3 & 0.2 \end{bmatrix}^{T}$$

$$D = \left(\begin{bmatrix} 0.1 & 0.2 & 0.25 \\ 0.15 & 0.4 & 0 \\ 0.1 & 0.3 & 0.2 \end{bmatrix}^{T}\right)^{-1}$$

$E:$ It can't be deduced based on the given information.

Problem 2. Let M, p and d be internal demand matrix, production vector and external demand vector. Then according to the Leontiev input-output model with open economy, we have

A

$$p = \left(\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - M\right)^{-1} \cdot d$$

B

$$p = \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - M\right) \cdot d$$

C

$$p = \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - M\right)^{-1} \cdot d$$

D

$$d = \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - M\right) \cdot p$$

E

$$p = \left(\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - M\right) \cdot d$$

Problem 3. In the Leontiev input-output model with closed economy ($d = 0$) we have:

A any eigenvector p of M provides valid production values,

B any eigenvector p of M associated with the eigenvalue 1 provides valid production values,

C $v = 0$ always provides valid production values,

D any unit eigenvector p of M associated with the eigenvalue 1 provides valid production values,

E any unit eigenvector p of M provides valid production values.

Problem 4. For the matrix $(I - M)^{-1}$:

A the diagonal entries are always ≥ 1,

B the diagonal entries are always ≥ 0,

C jth **column** says how much the production of each product has to be increased in order to satisfy the increase in demand on product j by $+1$,

D jth **row** says how much the production of each product has to be increased in order to satisfy the increase in demand on product j by $+1$,

E all entries are always ≥ 1.

Problem 5. Let

$$M = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.3 \end{bmatrix}.$$

A It describes an economy with independent sectors,

B there is no theoretically possible economy associated with M,

C $(I - M)^{-1} = I + M + M^2 + M^3 + ...$,

D $(I - M)^{-1} = \lim_{k\to\infty} M^k$,

E In closed economy the only production vector satisfying the Leontiev model with the internal production M is $p = 0$.

Problem 6. We rotate point $(0,0)$ in $\mathbb{R}^2$ by $\frac{\pi}{3}$ counterclockwise around point $(1,7)$.

A The coordinates of the point after the rotation are the upper two entries of the vector

$$v = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\pi/3) & -\sin(\pi/3) & 0 \\ \sin(\pi/3) & \cos(\pi/3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

B The coordinates of the point after the rotation are the upper two entries of the vector

$$v = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\pi/3) & -\sin(\pi/3) & 0 \\ \sin(\pi/3) & \cos(\pi/3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

C The coordinates of the point after the rotation are the upper two entries of the vector

$$v = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\pi/3) & -\sin(\pi/3) & 0 \\ \sin(\pi/3) & \cos(\pi/3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

divided by the bottom entry.

D The coordinates of the point after the rotation are the upper two entries of the vector

$$v = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\pi/3) & -\sin(\pi/3) & 0 \\ \sin(\pi/3) & \cos(\pi/3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

divided by the bottom entry.

E The coordinates of the point after the rotation are equal to the entries of the vector

$$v = \begin{bmatrix} 1 & -1 \\ 0 & -7 \end{bmatrix} \begin{bmatrix} \cos(\pi/3) & -\sin(\pi/3) \\ \sin(\pi/3) & \cos(\pi/3) \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Problem 7. Rotation in 3D around the y-axis by $\frac{\pi}{2}$, so that the positive z go in the direction of negative x is associated with the matrix

$$A = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\pi/2) & -\sin(\pi/2) & 0 \\ 0 & \sin(\pi/2) & \cos(\pi/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} \cos(\pi/2) & 0 & \sin(\pi/2) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\pi/2) & 0 & \cos(\pi/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad E : \begin{bmatrix} \cos(\pi/2) & -\sin(\pi/2) & 0 & 0 \\ \sin(\pi/2) & \cos(\pi/2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 8. The perspective projection of the point $(1,2,3)$ onto the $z=0$ plane from the center of perspective $(0,0,777)$ is associated with the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{777} & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{777} & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{777} & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{777} & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} \qquad E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 777 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

Problem 9. The homogeneous coordinates of the point (x, y, z) are equal to

$$v = \begin{bmatrix} 7 \\ 3 \\ 5 \\ 7 \end{bmatrix}.$$

Then

A The homogeneous coordinates of this point are also equal to $\frac{1}{7}v$.

B The homogeneous coordinates of this point are also equal to $2v$.

C The standard coordinates of the point (x, y, z) are equal to $(7, 3, 5)$

D Something went wrong in the calculations: it should always be 1 in the bottom of the homogeneous coordinates.

E The standard coordinates of the point (x, y, z) are equal to $(1, \frac{3}{7}, \frac{5}{6})$

Problem 10. The least-squares solution to the equation

$$\underbrace{\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}}_{=A} x = \underbrace{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}_{=b}$$

A is the solution of the equation

$$\begin{bmatrix} 2 & 2 \\ 2 & 5 \end{bmatrix} x = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

B is the solution of the equation $Ax = Proj_{Col(A)}b$

C always exists

D is equal to $\hat{x} = (AA^T)A^T b$

E is the element in $Col(A)$ that is the closest to b.

Problem 11. In the previous problem the solution is

$$\hat{x} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}.$$

The least-squares error is

A equal to $\|\hat{x} - b\|$, where $\|v\|$ is the length of the vector v,

B equal to

$$\left\| \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} - b \right\|,$$

C is impossible to calculate based on provided information

D equal to $\sqrt{5}$

E equal to $\sqrt{6}$.

Problem 12. Curve fitting of points $(0,0)$, $(1,2)$ and $(2,3)$ using the template ax^2+b:

A is associated with the least-squares solution of

$$\begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$$

B is associated with the least-squares solution of

$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$$

C is associated with the solution of

$$\begin{bmatrix} 1 & 3 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

D does not exist since point $(0,0)$ belongs to any linear space

E cannot yield a smaller least-squares error than when using the template ax^2+bx+c.

Problem 13. Consider points $(0,1)$, $(1,2)$ and $(2,5)$ belonging to the curve $y=x^2+1$. Then

A it is impossible to perform a least-squares line fitting, since the points belong to a parabola,

B the template ax^2+b can't be used in this case because it would yield a 0 least-squares error for $a=b=1$,

C the template ax^2+b yields a 0 least-squares error for $a=b=1$

D it is impossible to get a 0 least squares error for the given points using line-fitting.

E it is possible to get a 0 least squares error for the given points using line-fitting **after removing one point** (thus, working with only two points).

Problem 14. Suppose that 4 teams play a number of games with the following results

- Team 1 beats team 2 by 7 points
- Team 1 loses to team 3 by 8 points
- Team 2 ties with team 3
- Team 2 beats team 3 by 15 points
- Team 2 beats team 4 by 5 points
- Team 3 loses to team 4 by 1 points

The Massey method team rankings for the above teams are represented by a vector x, which is

A a **least-squares** solution to

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 4 & -2 & -1 \\ -1 & -2 & 4 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} x = \begin{bmatrix} -1 \\ 13 \\ -8 \\ -4 \end{bmatrix}$$

B a **least-squares** solution to

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} x = \begin{bmatrix} 7 \\ -8 \\ 0 \\ 15 \\ 5 \\ -1 \end{bmatrix}$$

C a solution to

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 4 & -1 & -1 \\ -1 & -2 & 4 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} x = \begin{bmatrix} -1 \\ 13 \\ -8 \\ -4 \end{bmatrix}$$

D a solution to

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 4 & -2 & -1 \\ -1 & -2 & 4 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix} x = \begin{bmatrix} -1 \\ 13 \\ -8 \\ 0 \end{bmatrix}$$

E a solution to

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 4 & -2 & -1 \\ -1 & -2 & 4 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} x = \begin{bmatrix} -1 \\ 13 \\ -8 \\ 1 \end{bmatrix}$$

Problem 15. If the Massey matrix equation for a set of teams is

$$\begin{bmatrix} 3 & -1 & -2 & 0 \\ -1 & 5 & -2 & -2 \\ -2 & -2 & 4 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} x = \begin{bmatrix} 6 \\ 2 \\ 10 \\ 0 \end{bmatrix}$$

then

A There are 4 teams

B Team 4 played 1 game

C Team 1 played 1 game against team 2

D Team 2 played 3 games in total

E In total there were 7 games played

Problem 16. Let T be a regular transition matrix. Then

A Each column of T, treated as a vector, has length 1

B Each column of T has the sum of its entries equal to 1

C Each column is a probability vector

D One of its eigenvalues is equal to 1

E Each row is a probability vector

Problem 17. Let T be a regular transition matrix. Then

A any eigenvector of T corresponding to the eigenvalue 1 is a steady state of the Markov chain generated by T

B any unit eigenvector of T corresponding to the eigenvalue 1 is a steady state of the Markov chain generated by T

C any probability eigenvector of T corresponding to the eigenvalue 1 is a steady state of the Markov chain generated by T

D if v is the steady state then $Tv = v$

E if v is the steady state then $Tx = v$ for any vector x

Problem 18. Let T be a transition matrix. Then

A if T is regular then it has a steady state v

B if T is regular then $\lim_{k\to\infty} T^k x = v$, where x is any vector and v is the steady state

C if T is not regular then it may still have a steady state

D if T is not regular then point B is always false. In other words, there is no chance to find a transition matrix T, that is not regular, but it still has a steady state and every other state converges to the that steady state

E $\lim_{k\to\infty} T^k x = v$, where x is any vector and v is the steady state

Problem 19. Suppose that we have 3 states such that

- all residents of state 1 remain in state 1
- half of the residents of state 2 moves to state 1 and the other half moves to state 3
- all residents of state 3 move to state 2

A The transition matrix T for this process is

$$\begin{bmatrix} 1 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$$

B The transition matrix for this process is

$$\begin{bmatrix} 1 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \\ 0 & \frac{1}{2} & 0 \end{bmatrix}^T$$

C T has a steady state equal to $[1, 0, 0]^T$

D T is a regular transition matrix

E we have $\lim_{k \to]infty} T^k [1/3, 1/3, 1/3]^T = [1, 0, 0]^T$.

Problem 20. Consider the internet with four pages $P1$, $P2$, $P3$ and $P4$. Suppose that the links are ass follows: $P1 \to P2, P3$, $P2 \to P3$, $P3 \to P2, P4$ and $P4 \to P1$. Then

A the transition matrix associated with the Google Page Rank for this internet is

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \end{bmatrix}$$

B the transition matrix associated with the Google Page Rank for this internet **is not** regular

C not only is point B false, but also **every** transition matrix associated with the Google Page Rank for **any** internet is **always** regular

D the ranking of each page belongs to the interval $[0, 1]$

E suppose that we remove all links of all pages of this internet; then the Google Page Ranking is equal for all 4 pages

Problem 21. Singular value decomposition of matrix A is a factorization $A = U\Sigma W$ and

A the columns of U are unit eigenvectors of AA^T corresponding to eigenvalues arranged in a nonincreasing order

B the columns of W are unit eigenvectors of $A^T A$ corresponding to eigenvalues arranged in a nonincreasing order

C the only nonzero entries of Σ are the nonzero eigenvalues of AA^T arranged in a nonincreasing order on the diagonal

D singular values are the square roots of the nonzero eigenvalues of AA^T

E this sentence is false

Problem 22. Let

$$M=\begin{bmatrix}1 & 2 & 0\\ 0 & 1 & -1\end{bmatrix}.$$

Then the SVD of M is equal to

$$A=\begin{bmatrix}2 & 1\\ 1 & -2\end{bmatrix}\begin{bmatrix}\sqrt{6} & 0 & 0\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}2 & 5 & -1\\ 1 & 0 & 2\\ -2 & 1 & 1\end{bmatrix},\quad B=\frac{1}{5}\begin{bmatrix}2 & 1\\ 1 & -2\end{bmatrix}\begin{bmatrix}\sqrt{6} & 0 & 0\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\frac{2}{\sqrt{6}} & \frac{5}{\sqrt{6}} & -\frac{1}{\sqrt{6}}\\ 1 & 0 & 2\\ -2\frac{\sqrt{5}}{\sqrt{6}} & \frac{\sqrt{5}}{\sqrt{6}} & \frac{\sqrt{5}}{\sqrt{6}}\end{bmatrix},$$

$$C=\frac{1}{5}\begin{bmatrix}-2 & 1\\ -1 & -2\end{bmatrix}\begin{bmatrix}\sqrt{6} & 0 & 0\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}-\frac{2}{\sqrt{6}} & -\frac{5}{\sqrt{6}} & \frac{1}{\sqrt{6}}\\ 1 & 0 & 2\\ -2\frac{\sqrt{5}}{\sqrt{6}} & \frac{\sqrt{5}}{\sqrt{6}} & \frac{\sqrt{5}}{\sqrt{6}}\end{bmatrix},\quad D=\begin{bmatrix}2 & 1\\ 1 & -2\end{bmatrix}\begin{bmatrix}\sqrt{6} & 0 & 0\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}2 & 5 & -1\\ 1 & 0 & 2\\ -2 & 1 & 1\end{bmatrix}^T,$$

$$D=\frac{1}{5}\begin{bmatrix}2 & 1\\ 1 & -2\end{bmatrix}\begin{bmatrix}\sqrt{6} & 0 & 0\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\frac{2}{\sqrt{6}} & \frac{5}{\sqrt{6}} & -\frac{1}{\sqrt{6}}\\ 1 & 0 & 2\\ -2\frac{\sqrt{5}}{\sqrt{6}} & \frac{\sqrt{5}}{\sqrt{6}} & \frac{\sqrt{5}}{\sqrt{6}}\end{bmatrix}^T.$$

Problem 23. Let A be a centered data matrix such that

$$A=[u_1 \quad u_2]\begin{bmatrix}\sigma_1 & 0 & 0\\ 0 & \sigma_2 & 0\end{bmatrix}V^T$$

is its SVD. Then

A the total variance of the data is equal to $\sigma_1+\sigma_2$

B the percentage of the variance of the data in the direction u_2 is equal to $\frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2}$

C vector u_1 defines the direction in which the data vary the most

D $\|u_1\|=\|u_2\|^2$

E $\sigma_1\geq\sigma_2$

Problem 24. Let A be a centered data matrix such that

$$A=U\Sigma V^T$$

is its SVD. Suppose that jth eigenvalue of AA^T is nonzero. Then

A If u_j is the jth column of U then Au_j is the jth column of V

B If u_j is the jth column of U then A^Tu_j is the jth column of V

C If u_j is the jth row of U then A^Tu_j is the jth column of V

D If u_j is the jth column of U then $\frac{A^Tu_j}{\|u_j\|}$ is the jth column of V

E If u_j is the jth column of U then $\frac{A^Tu_j}{\|A^Tu_j\|}$ is the jth column of V

Problem 25. Let

$$A=\begin{bmatrix}2 & 1 & -3 & 23 & 1 & 0 & -12 & 8 & \pi & \sqrt{1}\\ 4 & -12 & 8 & 0 & 93 & 0 & 55 & 99 & 1 & -1\end{bmatrix}.$$

Then, in practice, to find the SVD of A one **necessarily** needs to (maybe among other things)

A find 2 eigenvalues of AA^T

B find 2 eigenvectors of AA^T

C find 10 eigenvalues of A^TA

D find 10 eigenvectors of A^TA

E it would be **much** more tedious to perform the SVD for A^T