1 Example Midterm

The following problems are similar[1] *to what can be expected on the actual midterm. If you spot any mistakes, please let me know.*

Problem 1.1.

(A) Find the inverse of

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 1 & -1 \end{bmatrix}.$$

(B) Find **all** the eigenvalues and **one** eigenvector of

$$B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$

(C) For A from (A) solve

$$A\mathbf{x} = \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}.$$

Problem 1.2. Suppose an economy has three sectors producing products P1, P2 and P3 respectively.

- It takes 0.2 units of P1, 0.15 units of P2 and 0.1 units of P3 to produce 1 unit of P1.
- It takes 0.1 units of P1, 0.05 units of P2 and 0.12 units of P3 to produce 1 unit of P1.
- It takes 0.14 units of P1 and 0.08 units of P3 to produce 1 unit of P3.

Suppose that we want to satisfy the external demand of 401 units of P1, 104 units of P2 and 14 units of P3.

(A) Write down the consumption matrix M and the external demand vector $\mathbf{d}$.

(B) Write the main equation of the Leontief Input-Output model that showcases the balance between total production and internal and external demand.

(C) Express the required total production $\mathbf{p}$ with a formula using M and $\mathbf{d}$.

Problem 1.3. Suppose that M is the consumption matrix for an economy consisting of 3 products: cars, cats and milk (in this order!). Suppose further that the *marginal production response matrix* $(I - M)^{-1}$ is given by

$$(I - M)^{-1} = \begin{bmatrix} 1.0206 & 0.002 & 0.1022 \\ 0.0634 & 1.0008 & 0.00467 \\ 0.037 & 0.2 & 1.093 \end{bmatrix}.$$

How much should the production of cars, cats and milk be increased in order to satisfy the increase in the demand of cars by $+2$?

[1] similar$\neq$ the same (in particular I reserve myself the right to make 1 problem completely different to what is presented here)

Problem 1.4. Suppose an economy has two sectors producing products P1 and P2 respectively.

- It takes 0.5 units of P1 and 0.2 units of P2 to produce 1 unit of P1.
- It takes x units of P2 to produce 1 unit of P2.

Find all x for which this scenario is economically sensible. That means the following need to be satisfied:

1. There exists a solution for any external demand.
2. Diagonal entries of the marginal production response matrix are not smaller than 1.
3. Other entries are not smaller than 0.

Problem 1.5. Consider the tetrahedron with vertices $(0,1,2)$, $(1,0,3)$, $(1237,1230913480,12831)$ and $(\pi,e,401)$. We want to translate it so that the vertex $(0,1,2)$ is in the origin, then rotate it around the line $z=1, y=7$ by $\pi/3$ radians in the direction that positive part of OY goes in the direction of negative OZ. Finally we want to display the xy-perspective projection of the obtained set from the center of perspective $(0,0,401)$. This whole procedure can be represented by a sequence of operations. Write this sequence (in proper order). Explicitly state the matrix for each individual operation but don't calculate the final product.

Problem 1.6. Find the vector in

$$\text{span}\left\{\begin{bmatrix}1\\2\end{bmatrix}\right\} \quad \text{closest to} \quad \begin{bmatrix}5\\5\end{bmatrix}.$$

Find the least-squares error.

Problem 1.7. Find the least-squares curve $f(x)=ax+b\sqrt{x}$ for the points $(1,2)$, $(2,4)$ and $(4,5)$. What quantity is minimized by this curve? Do not perform the final calculation. Just find the formula $(a,b)=$ .....

Problem 1.8. Which of the following function templates are admissible in the least-squares method of curve fitting?

- $f(x)=ax^2+bx^c$
- $a\pi^{ex}+bx$
- $f(x)=asin(x)+b\tan(x)$
- $f(x)=asin(bx)+c$

Problem 1.9. Suppose four teams play a number of games with the following results:

- Team 1 beats Team 2 by 7 points.
- Team 1 loses to Team 3 by 8 points.
- Team 2 ties with Team 3.
- Team 2 beats Team 3 by 15 points.
- Team 2 beats Team 4 by 5 points.
- Team 3 loses to Team 4 by 1 point. Using the Massey method find the team rankings (don't do the final calculation, just express the rankings by a propper formula; all the involved matrices must be explicitly stated though).