On 81 symplectic resolutions of a 4-dimensional quotient by a group of order 32

Maria Donten-Bury and J. A. Wiśniewski
ongoing joint project

Uniwersytet Warszawski

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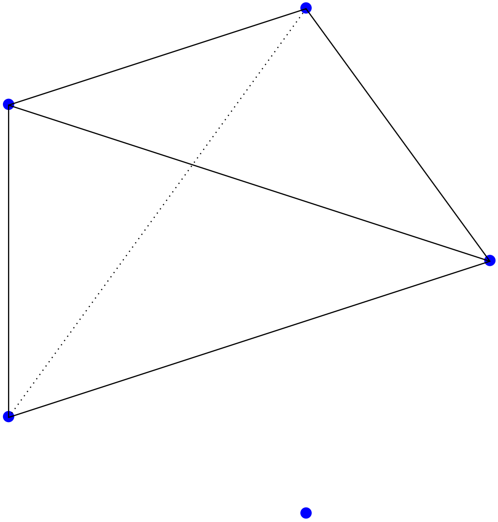
Plan

1. Classical geometry
2. 81 resolutions
3. A Kummer 4-fold (with MD-B and G. Kapustka)

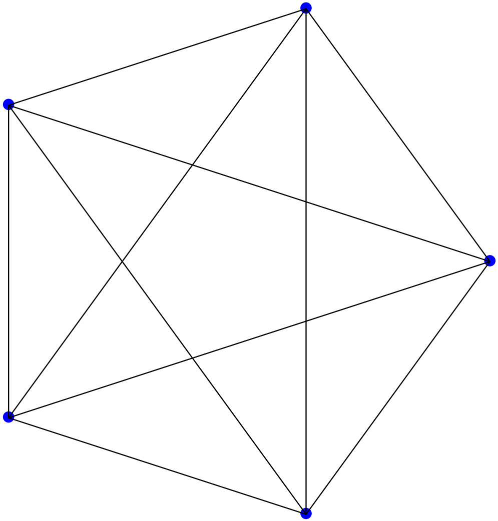
This polytope comes with different names (according to Wikipedia):

- Dispentachoron
- Rectified 5-cell
- Rectified pentachoron [RAP]
- Rectified 4-simplex
- Ambopentachoron

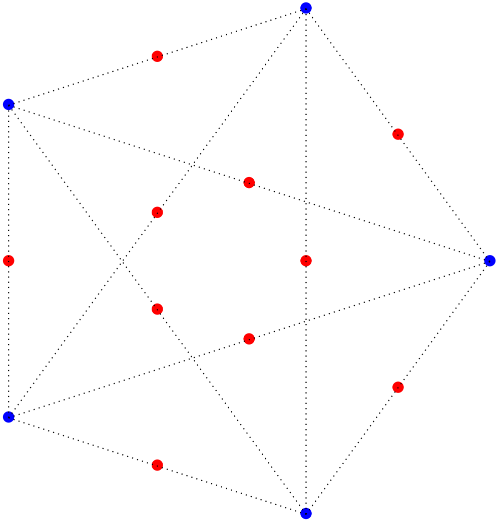
3 dimensional simplex



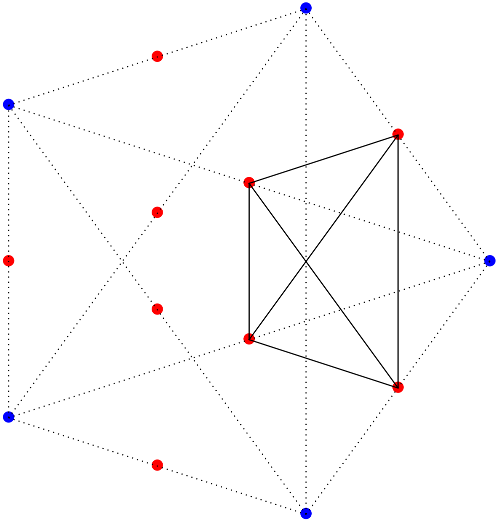
4 dimensional simplex



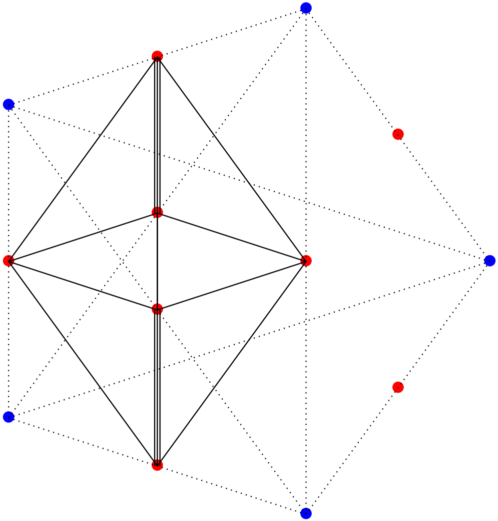
vertices of RAP



5 simplicial faces of RAP



5 octahedral faces of RAP



$\mathbb{P}^2$ blown up at 4 points

We consider a surface $\mathbb{P}^2_4$ obtained by blowing up the complex plane $\mathbb{P}^2$ at generic 4 points.

$\mathbb{P}^2_4$ has 6 more (-1) curves which come from the lines passing through the pairs of points which we blow up.

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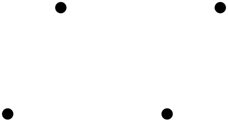

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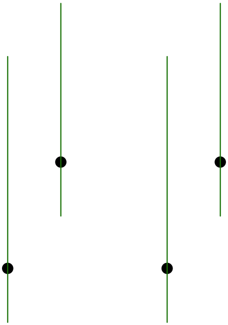

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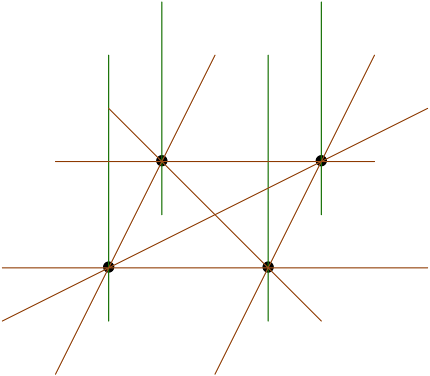

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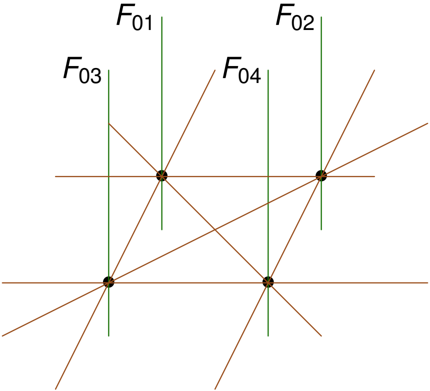

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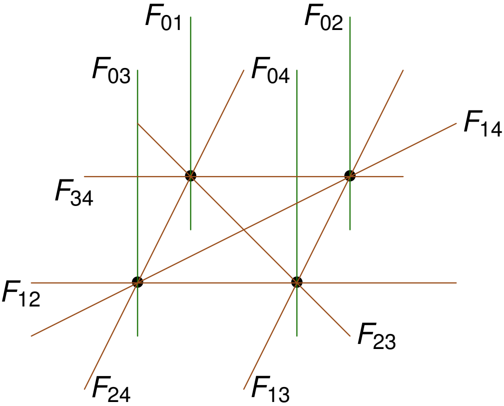

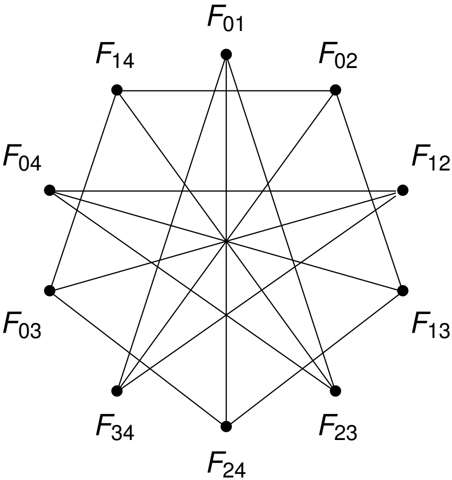

(-1)-curve are dots, incidence denoted by line segments; result: Petersen graph.

divisors on $\mathbb{P}^2_4$

Consider 5-dimensional $\mathbb{R}$-vector space N with a basis $e_0, \ldots, e_4$. For $0 \le i < j \le 4$ we set $f_{ij} = (e_i + e_j)/2$.

- we can identify $N := Pic(\mathbb{P}^2_4) \otimes \mathbb{R}$ with f_{ij} classes of (-1)-curves
- the cone of effective divisors $\mathrm{Eff}(\mathbb{P}^2_4) = \sum_{i,j} \mathbb{R}_{\ge 0} \cdot f_{ij}$ has 5 simplicial facets associated to contractions to $\mathbb{P}^2$,
- the total coordinate ring

$$\mathcal{R}_{\mathbb{P}^2_4} = \bigoplus_{[D] \in \mathrm{Pic}\, \mathbb{P}^2_4} \Gamma(\mathbb{P}^2_4, \mathcal{O}(D))$$

is generated by the sections x_{ij} associated to f_{ij}'s and

$$\mathcal{R}_{\mathbb{P}^2_4} = \mathbb{C}[x_{ij} : 0 \le i < j \le 4]/(x_{pq}x_{rs} - x_{pr}x_{qs} + x_{ps}x_{qr})$$

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algebraic torus action and $\mathbb{Z}^r$ grading

Suppose that

$$\mathcal{R} = \bigoplus_{\mu \in \mathbb{Z}^r} \Gamma^\mu$$

is a graded finitely generteed $\mathbb{C}$-algebra, which means that

$$\Gamma^\mu \cdot \Gamma^{\mu'} \subset \Gamma^{\mu+\mu'}$$

Then the algebraic torus $\mathbb{T} = (\mathbb{C}^*)^r$ with coordinates $t = (t_1, \dots t_r)$ acts on $\mathcal{R}$:

$$\mathbb{T} \times \Gamma^\mu \ni (t, f) \longrightarrow t_1^{\mu_1} \cdots t_r^{\mu_r} \cdot f$$

Γ^μ are eigenspaces of the action of $\mathbb{T}$; in particular Γ^0 is the space of invariants of the action

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For $f \in \Gamma^{\mu}$ we take invariant fractions

$$\mathcal{R}_f^0 = \{u/f^m : m \geq 0, u \in m\mu\}$$

then $\mathcal{R}_f^0$ defines a more refined set of orbits of action of $\mathbb{T}$.

Idea: given the ideal $I = (f_1, \ldots f_s) \triangleleft \mathcal{R}$ generated by homogeneous $f_j \in \Gamma^{\mu^j}$ the sets associated to $\mathcal{R}_f^0$ for homogeneous $f \in I$ can be patched together to form a space of orbits.

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Take the ring

$$\mathcal{R}_{\mathbb{P}^2_4} = \bigoplus_{[D] \in \operatorname{Pic} \mathbb{P}^2_4} \Gamma(\mathbb{P}^2_4, \mathcal{O}(D))$$

and for a divisor $D \in \operatorname{Eff}(\mathbb{P}^2_4)$ take

$$I = \sqrt{(\Gamma(X, \mathcal{O}(mD)) : m \gg 0)}$$

The GIT quotient of the ring $\mathcal{R}_{\mathbb{P}^2_4}$ depends on the choice of the divisor D.

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The Mumford quotient depends on the choice of $[D] \in \mathrm{Eff}(\mathbb{P}^2_4)$.

In fact $\mathrm{Eff}(\mathbb{P}^2_4)$ is divided by hyperplanes into 76 chambers which are associated to different quotients

isomorphism class of quotient	number of chambers
$\mathbb{P}^2_4$	one, $\mathrm{Nef}(\mathbb{P}^2_4)$
$\mathbb{P}^2_3$	ten
$\mathbb{P}^2_2$	thirty
$\mathbb{P}^2_1$	twenty
$\mathbb{P}^2$	five [$\rightarrow$ simplicial facets $\mathrm{Eff}(\mathbb{P}^2_4)$]
$\mathbb{P}^1 \times \mathbb{P}^1$	ten

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halftime summary

- take a projective variety X such that $\mathrm{Pic}(X) = \mathbb{Z}^r$, e.g. $X = \mathbb{P}^2_4$
- construct its total coordinate ring

$$\mathcal{R}_X = \bigoplus_{[D] \in \mathrm{Pic}(X)} \Gamma(X, \mathcal{O}(D))$$

  suppose $\mathcal{R}_X$ is finitely generated $\mathbb{C}$-algebra
- the grading in $\mathrm{Pic}(X)$ determines an action of a torus $\mathbb{T}$
- Mumford's GIT allows to recover X as a quotient of $\mathcal{R}_X$ by the action of $\mathbb{T}$; same concerns some birational modifications of X

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Plan

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3. A Kummer 4-fold (with MD-B and G. Kapustka)

Let V be a 4-dimensional $\mathbb{C}$ vector space with coordinates $(x_1, \ldots, x_4)$ and the symplectic form $dx_1 \wedge dx_3 + dx_2 \wedge dx_4$.

The following reflections preserve this form

$$T_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

the reflections

$$T_1 = \begin{pmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}$$

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$$T_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

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$$T_3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

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$$T_4 = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}$$

The group G generated by reflections $T_0, \ldots, T_4$ has 32 elements in 17 conjugacy clases:

- $[I]$ and $[-I]$
- 5 classses of reflection $\pm T_i$
- 10 classes of $\pm$ elements of order 4

Moreover $[G, G] = \langle -I \rangle$ and $Ab(G) = G/[G, G] = \mathbb{Z}_2^4$

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quotient and resolution

- Bellamy and Schedler: there exists a symplectic resolution X of V/G (non-constructive proof following Namikawa and Ginzburg-Kaledin smoothing)
- Kaledin: the resolution should have 5 divisors contracted to 5 surfaces of A_1 singularities and one exceptional fiber with 11 components of dimension 2 (McKay correspondence)
- Wierzba and W: all resolutions of V/G differ by Mukai flops ($\mathbb{P}^2$ flopped to its dual)
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the action of $Ab(G)$ on $V/[G,G]$

The ring of invariants of $[G,G] = \langle -I \rangle$ is generated by quadratic forms, S^2V^*. The quadratic invariants decompose into ± 1 eigenspaces of $Ab(G)$:

	eigenfunction	T_0	T_1	T_2	T_3	T_4
$\phi_{01} =$	$-2(x_1x_4 + x_2x_3)$	$-$	$-$	$+$	$+$	$+$
$\phi_{02} =$	$2\sqrt{-1}(-x_1x_4 + x_2x_3)$	$-$	$+$	$-$	$+$	$+$
$\phi_{03} =$	$2\sqrt{-1}(x_1x_2 + x_3x_4)$	$-$	$+$	$+$	$-$	$+$
$\phi_{04} =$	$2(-x_1x_2 + x_3x_4)$	$-$	$+$	$+$	$+$	$-$
$\phi_{12} =$	$2(x_1x_3 - x_2x_4)$	$+$	$-$	$-$	$+$	$+$
$\phi_{13} =$	$-x_1^2 - x_2^2 + x_3^2 + x_4^2$	$+$	$-$	$+$	$-$	$+$
$\phi_{14} =$	$\sqrt{-1}(x_1^2 + x_2^2 + x_3^2 + x_4^2)$	$+$	$-$	$+$	$+$	$-$
$\phi_{23} =$	$\sqrt{-1}(-x_1^2 + x_2^2 - x_3^2 + x_4^2)$	$+$	$+$	$-$	$-$	$+$
$\phi_{24} =$	$x_1^2 - x_2^2 - x_3^2 + x_4^2$	$+$	$+$	$-$	$+$	$-$
$\phi_{34} =$	$2(x_1x_3 + x_2x_4)$	$+$	$+$	$+$	$-$	$-$

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$\phi_{12} =$	$2(x_1x_3 - x_2x_4)$	$+$	$-$	$-$	$+$	$+$
$\phi_{13} =$	$-x_1^2 - x_2^2 + x_3^2 + x_4^2$	$+$	$-$	$+$	$-$	$+$
$\phi_{14} =$	$\sqrt{-1}(x_1^2 + x_2^2 + x_3^2 + x_4^2)$	$+$	$-$	$+$	$+$	$-$
$\phi_{23} =$	$\sqrt{-1}(-x_1^2 + x_2^2 - x_3^2 + x_4^2)$	$+$	$+$	$-$	$-$	$+$
$\phi_{24} =$	$x_1^2 - x_2^2 - x_3^2 + x_4^2$	$+$	$+$	$-$	$+$	$-$
$\phi_{34} =$	$2(x_1x_3 + x_2x_4)$	$+$	$+$	$+$	$-$	$-$

$S^2 V^* \simeq \Lambda^2 W^*$

The labeling of functions ϕ_{rs} indicates an isomorphism between $S^2 V^*$ and $\Lambda^2 W^*$ where W is a 5-dimensional space with coordinates $t_0, \dots t_4$:

$$F_{ij} \leftrightarrow t_i \wedge t_j$$

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We define a subring $\mathcal{R}_G$ in

$$\mathbb{C}[V] \otimes \mathbb{C}[\mathbb{T}_W] = \mathbb{C}[x_1, x_2, x_3, x_4, t_0^{\pm 1}, t_1^{\pm 1}, t_2^{\pm 1}, t_3^{\pm 1}, t_4^{\pm 1}]$$

generated by

- $\phi_{ij} \cdot t_i \cdot t_j$ for $0 \leq i < j \leq 4$
- t_i^{-2} for $i = 0, \ldots, 4$

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total coordinate ring

- $\mathbb{C}[V]^G \simeq \mathcal{R}_G^{\mathbb{T}_W}$
- some GIT quotients of Spec $\mathcal{R}_G$ are smooth and provide desingularisation of Spec $\mathbb{C}[V]^G$
- $\mathcal{R}_G$ is the total coordinate ring of one (hence every) symplectic desingularisation of V/G
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\phi_{14}\phi_{23} + \phi_{13}\phi_{24} - \phi_{12}\phi_{34} &= 0\\
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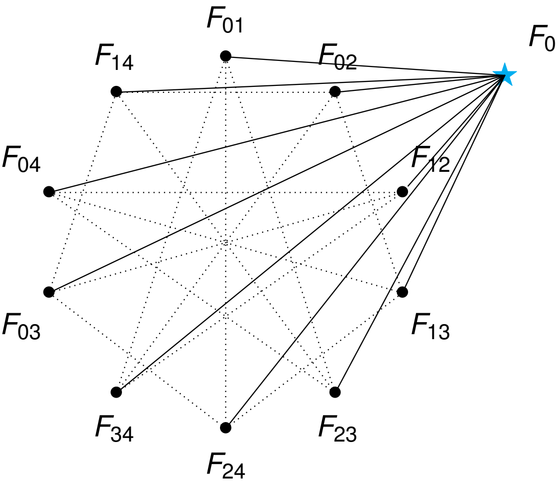
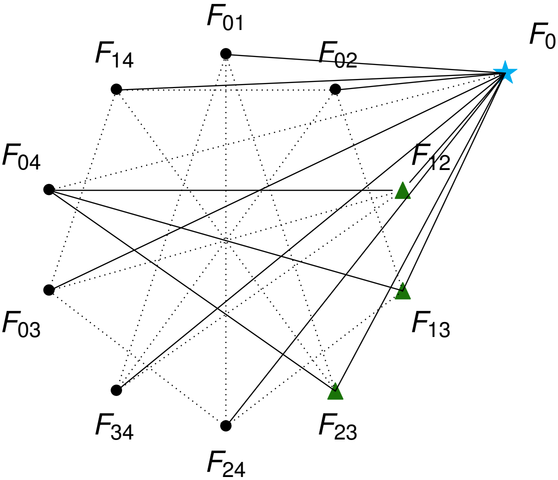
the incidence of components – central chamber

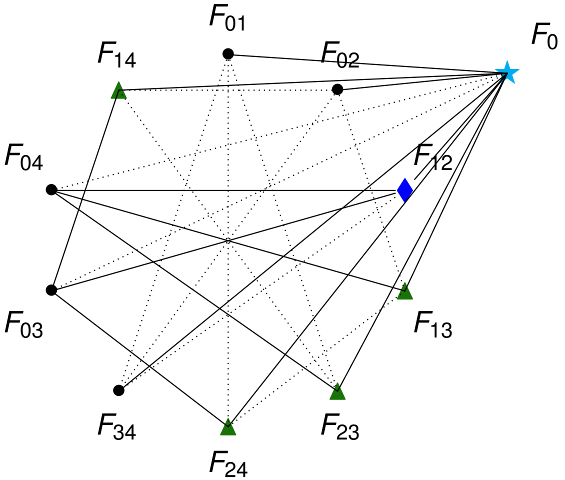


$F_0 = \mathbb{P}^2_4$, $F_{ij} = \mathbb{P}^2$, solid (dotted) lines = intersection in lines (pts)

first flop, 10 chambers

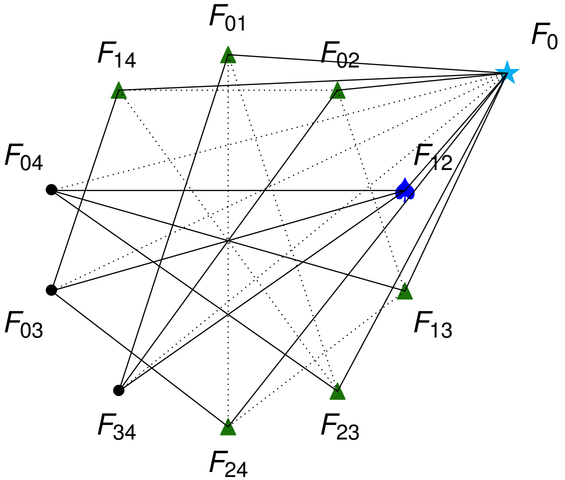


here $F_0 = \mathbb{P}^2_3$, and $\bullet = \mathbb{P}^2$, $\blacktriangle = \mathbb{P}^2_1$



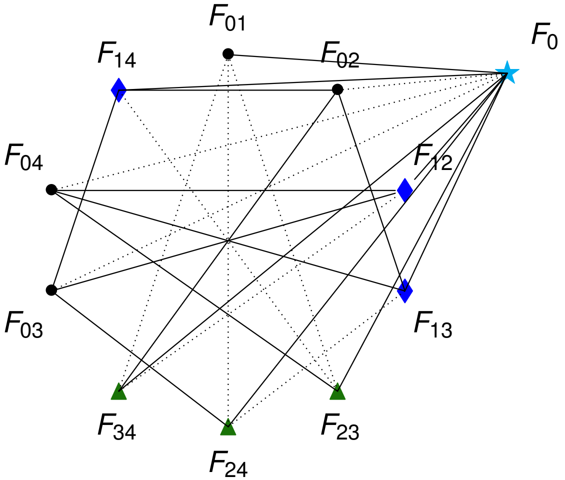
here $F_0 = \mathbb{P}^2_2$, and $\bullet = \mathbb{P}^2$, $\blacktriangle = \mathbb{P}^2_1$, $\blacklozenge = \mathbb{P}^2_2$

third flop, first option, 10 chambers



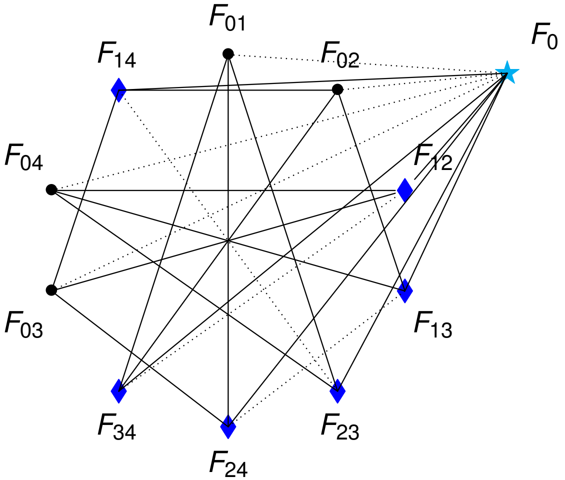
$F_0 = \mathbb{P}^1 \times \mathbb{P}^1$, ▲ $= \mathbb{P}^2_1$, and ♠ $= \mathbb{P}^2$ blown up in 3 collinear pts

third flop, second option, 20 chambers



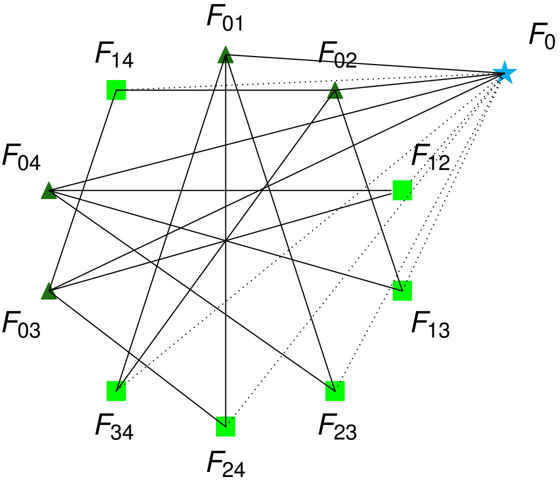
here $F_0 = \mathbb{P}^2_1$, and $\bullet = \mathbb{P}^2$, $\blacktriangle = \mathbb{P}^2_1$, $\blacklozenge = \mathbb{P}^2_2$

fourth flop, 5 chambers

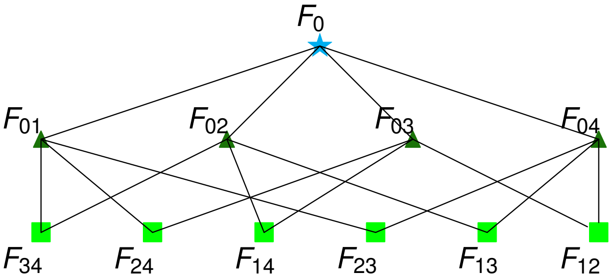


here $F_0 = \mathbb{P}^2$, and $\bullet = \mathbb{P}^2$, $\blacktriangle = \mathbb{P}^2_1$, $\blacklozenge = \mathbb{P}^2_2$

fourth flop, 5 outer chambers



here $F_0 = (\mathbb{P}^2)^\vee$, and ▲ $= \mathbb{P}^2_1$, ■ $= \mathbb{P}^1 \times \mathbb{P}^1$



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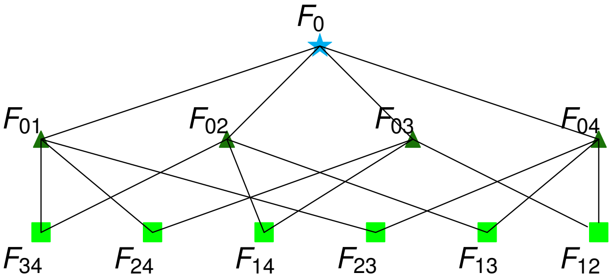

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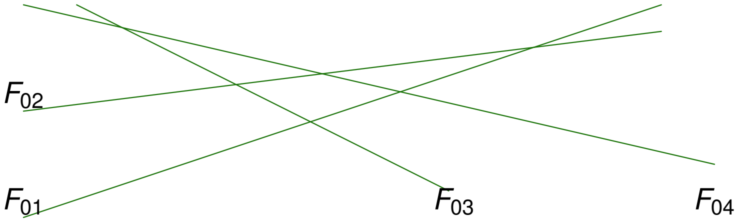

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- the ring of invariants $\mathbb{C}[V]^G$ defines quotient singularity V/G
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Suppose that $\varphi : X \to V/G$ is a resolution with exceptional divisor $\sum_i E_i$.

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Plan

1. Classical geometry
2. 81 resolutions
3. A Kummer 4-fold (with MD-B and G. Kapustka)

The group G can be presented as generated by another set of reflections in $Sp(4, \mathbb{Z}[i])$

more reflections

$$T_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1+i & 1 & 0 \\ 1-i & 0 & 0 & -1 \end{pmatrix}$$

more reflections

$$T_1 = \begin{pmatrix} 1 & 0 & 0 & -1-i \\ 0 & -1 & 1+i & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

more reflections

$$T_2 = \begin{pmatrix} 1 & -1+i & 0 & -1-i \\ -1-i & -1 & 1+i & 0 \\ 0 & -1+i & 1 & -1-i \\ 1-i & 0 & -1+i & -1 \end{pmatrix}$$

$$T_3 = \begin{pmatrix} i & 0 & 0 & 1-i \\ 1-i & -i & -1+i & 0 \\ 0 & -1-i & i & 1-i \\ 1+i & 0 & 0 & -i \end{pmatrix}$$

more reflections

$$T_4 = \begin{pmatrix} i & -1-i & 0 & 1-i \\ 0 & -i & -1+i & 0 \\ 0 & -1-i & i & 0 \\ 1+i & 0 & -1-i & -i \end{pmatrix}$$

Consider the action of G on C^4 where C is an elliptic curve admitting complex multiplication by i

- The reflections have fixed points consisting of 40 components.
- 256 points which are of order two on C^4 have bigger isotropy:
  - 16 have isotropy $= G$
  - 240 have isotropy $\mathbb{Z}_2 \times \mathbb{Z}_2$
- C^4/G admits symplectic resolution $X \to C^4/G$
- X is a new Kummer symplectic 4-fold
- The Poincare polynomial of X is the same as of $Hilb^2$ of K3

$$1 + 23t^2 + 276t^4 + 23t^6 + t^8$$

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