Boolean contact algebras

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Summary

The origins of Boolean contact algebras go back to the works of Leśniewski [8] on mereology and Leonard and Goodman [7] on the calculus of individuals on the one hand, and, on the other hand, the efforts of e.g. de Laguna [2], Tarski [12] and Whitehead [13] to use regions instead of points as the basic entity of geometry. A central role played the notion of "connection" (or "contact") of regions, which, in its simplest form, is a reflexive and symmetric relation C among non-empty regions, satisfying an additional extensionality axiom [3]. In order to formalize mereological structures (which were, basically, complete Boolean algebras B without a smallest element) together with Whitehead's connection relation C, Clarke [1] proposed additional axioms, among them a compatibility axiom $a \leq b \iff \{u : aCu\} \subseteq \{u : bCu\}$ and a summation axiom $aC(u+v) \iff aCu$ or aCv. A subsequent development, the Region Connection Calculus (RCC) [10], additionally supposed that each proper non-zero region was connected to its complement. Boolean algebras of regular closed sets of regular T_1 spaces, together with Whitehead's connection $aCb \iff a \cap b \neq \emptyset$ served as standard models for these "connection algebras". In a separate development, proximities on power set algebras were investigated which exhibited many similarities to the mereo-topological contact relations [6, 9, 11]. Three major questions arose:

1. Does every RCC algebra have representation as a subalgebra of the algebra of all regular closed sets of a topological space with Whitehead's connection relation?

2. What are some of the algebraic constructions of Boolean contact algebras?

3. Can one define on every atomless Boolean algebra an RCC contact relation?

In the seminar, I will present a solution to Question 1 [4]. I will also exhibit various methods how Boolean contact algebras can be constructed, and will give several non-standard examples, the most striking one being a countable model of the Region Connection Calculus in which every proper region has infinitely many holes [5]. Finally, I will present some progress towards a solution to Question 3.

References

[1] Clarke, B. L. (1981). A calculus of individuals based on 'connection'. *Notre Dame Journal of Formal Logic*, 22:204–218.

[2] de Laguna, T. (1922). Point, line and surface as sets of solids. *The Journal of Philosophy*, 19:449–461.

[3] Düntsch, I. and Orłowska, E. (2000). A proof system for contact relation algebras. *Journal of Philosophical Logic*, 29:241–262.

[4] Düntsch, I. and Winter, M. (2003). A representation theorem for Boolean contact algebras. Research report CS-03-08, Department of Computer Science, Brock University.

[5] Düntsch, I. and Winter, M. (2004). Construction of Boolean contact algebras. Research report CS-04-01, Department of Computer Science, Brock University.

[6] Efremovic, V. (1952). The geometry of proximity I. *Mat Sbornik (New Series)*, 31:189–200. In Russian.

[7] Leonard, H. S. and Goodman, N. (1940). The calculus of individuals and its uses. *Journal of Symbolic Logic*, 5:45–55.

[8] Leśniewski, S. (1927 – 1931). O podstawach matematyki. 30–34.

[9] Naimpally, S. A. and Warrack, B. D. (1970). *Proximity Spaces*. Cambridge University Press, Cambridge.

[10] Randell, D. A., Cohn, A. G., and Cui, Z. (1992). Computing transitivity tables: A challenge for automated theorem provers. In Kapur, D., editor, *Proceedings of the 11th International Conference on Automated Deduction (CADE-11)*, volume 607 of *LNAI*, pages 786–790, Saratoga Springs, NY. Springer.

[11] Swarc, A. S. (1956). Proximity spaces and lattices. *Ucen. Zap. Ivanovsk Gos. Ped. Inst.*, 10:55–60. In Russian.

[12] Tarski, A. (1956). Foundation of the geometry of solids. In Woodger, J. H., editor, *Logic, Semantics, Metamathematics*, pages 24–29. Clarendon Press, Oxford. Translation of the summary of an address given by A. Tarski to the First Polish Mathematical Congress, Lwów, 1927.

[13] Whitehead, A. N. (1929). *Process and reality*. MacMillan, New York.