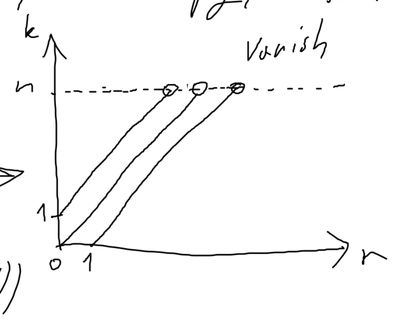
Recap: LCK - G-complexes

fix $n \geq 1$

fix Y - G-space; 1) $\forall_{H<G}$ Y^H - nonempty, arcwise connected, simple

$\pi_1(\ )$ = abelian and acts trivially on all homotopy groups

2,3,4)



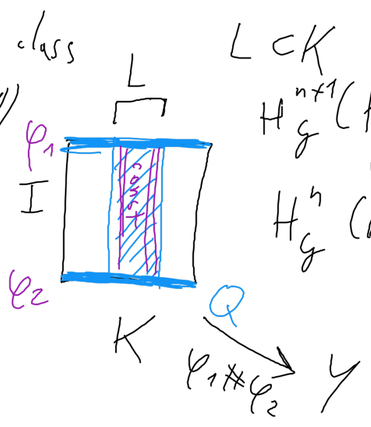
values $= H^k_G(K,L;\tilde{\omega}_r(Y))$

- have $\varphi: L \to Y$
- extend it to $K^n \cup L$ $\rightsquigarrow$ obstruction class
- $\gamma^{n+1}(\varphi) = [c_{\varphi_{ext}}] \in H^{n+1}_G(K,L;\tilde{\omega}_n(Y))$
- $\omega^n(\varphi_1,\varphi_2)$ $\quad \varphi_1,\varphi_2: K \to Y$

  $\| $

  $\lambda(\gamma^{n+1}(\varphi_1 \# \varphi_2))$ $\quad \varphi_1|_L = \varphi_2|_L$
- $\omega^n(\varphi_1,\varphi_2) + \omega^n(\varphi_2,\varphi_3) = \omega^n(\varphi_1,\varphi_3)$
- $k: (K',L') \xrightarrow{\text{cellular}} (K,L)$, and all coh. groups vanish as in

$L \subset K$

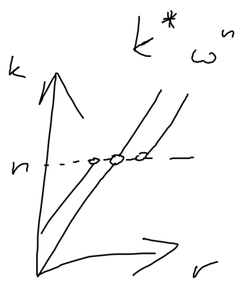


$H^{n+1}_G(K\times I, Q;\tilde{\omega}_n(Y))$

$\downarrow \| \wr$

$H^n_G(K,L;\tilde{\omega}_n(Y))$

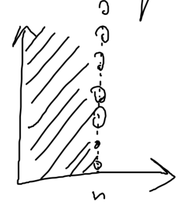
$k^*\omega^n(\varphi_1,\varphi_2) = \omega^n(\varphi_1\circ k, \varphi_2 \circ k)$

- 

; $\varphi: K \to Y$; equiv fun./equiv. homotopy rel. L $\overset{\text{bij}}{\longleftrightarrow}$ $H^n(K,L;\tilde{\omega}_n(Y))$

$\psi \longmapsto \omega^n(\varphi,\psi)$

$y_0 \in Y$: $\tilde{\omega}_q(Y,y_0) = 0$ for $q<n$ $\Rightarrow$ $H^k_G(K,L;\tilde{\omega}_q(Y)) = 0$



$L \subset K$; $L = \emptyset$

$x(\varphi_1) = \omega^k(\varphi_1, 0)$; $\quad \omega^k(\varphi_1,\varphi_2) = x(\varphi_1) - x(\varphi_2)$

$\omega^k(0,\varphi_1) + \omega^k(\varphi_1,0) = \omega^k(0,0) = 0 \in H^k$

$f_1 \quad f_2 \quad f_2 \quad f_3 \quad f_1 \quad f_3$

Def: Characteristic class of $\varphi: K \to Y$, y_0 is

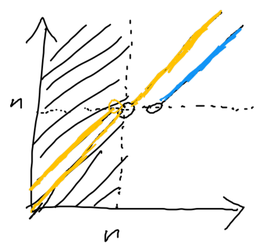
$x^n(\varphi) = \omega^n(f, \text{const}_{y_0}) \in H^n_G(K;\tilde{\omega}_n(Y))$

- if we have $k: K' \to K$; $\quad k^*(x^n(\varphi)) = x^n(\varphi\circ k)$

3.2) If $\boxed{H^r_G(K;\tilde{\omega}_r(Y)) = 0}$ for $r>n$,

then $\varphi_1,\varphi_2: K \to Y$ are homotopic iff $x^n(\varphi_1) = x^n(\varphi_2)$

Pf:



$K \to Y$ / homotopy rel. $\emptyset$ $\overset{\text{bij}}{\longleftrightarrow}$ $H^n_G(K;\tilde{\omega}_n(Y))$

$[\varphi] \longmapsto \omega^n(\varphi, \text{const}_{y_0}) = x^n(\varphi)$

const_{y_0}

$(K,\emptyset) \xrightarrow{j} (K,L)$

3.5) If $\varphi_1,\varphi_2: K \to Y$: $f|_L = g|_L \Rightarrow x^n(\varphi_1) - x^n(g) = j^*(\underbrace{\omega^n(\varphi_1,\varphi_2)}_{H^n_G(K,L;\tilde{\omega}_n(Y))}) \in H^n_G(K,\tilde{\omega}_n(Y))$

$\in H^n_G(K,L;\tilde{\omega}_n(Y))$

Pf:

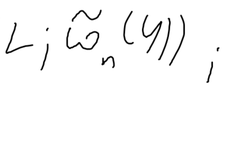
$x^n(\varphi_1) - x^n(\varphi_2) = \omega^n(\varphi_1,\text{const}_{y_0}) - \omega^n(\varphi_2,\text{const}_{y_0}) = \omega^n(\varphi_1,\underbrace{\text{const}_{y_0}}) + \omega^n(\underbrace{\text{const}_{y_0},\varphi_2}) = \omega^n(\varphi_1,\varphi_2) \in H^n_G(K;\tilde{\omega}_n(Y))$

$\varphi_1|_L = \varphi_2|_L$

3.3) $x^n(\varphi) \in H^n_G(L;\tilde{\omega}_n(Y))$; $\gamma^{n+1}(\varphi) = \delta^*(\underbrace{x^n(f)}_{H^n_G(L;\tilde{\omega}_n(Y))}) \in H^{n+1}_G(K,L;\tilde{\omega}_n(Y))$

$L \subset K$

3.4) If additionally



then an eq. $f: L \to Y$ has an extension to K

iff $x^n(f) \in \text{im}\, H^n_G(K;\tilde{\omega}_n(Y)) \to H^n_G(L;\tilde{\omega}_n(Y))$

$\downarrow \delta$

$H^{n+1}_G(K,L;\tilde{\omega}_n(Y))$

Pf) $\text{im} = \ker$ $\quad 0 = \gamma^{n+1}(f) = \delta(x^n(f))$

2.5)

Y - G-space, arcwise connected, $\boxed{\text{simple}}$ $\Rightarrow$ topo. groups $\rightsquigarrow$ $\pi_k(T^n) = \pi_k(S^1)^n$

$\Rightarrow$ simply connected spaces $\rightarrow$ dim 1 $\Rightarrow$ nothing

dim 2 $\xrightarrow{\text{comp}}$ S^2

dim 3 $\xrightarrow{\text{comp}}$ S^3

$k>1$: 0

$k=1$: $\mathbb{Z}^n$

vanishing $\tilde{\omega}_r(Y,y_0) \Leftrightarrow \pi_r(Y^H, y_0)$

$1 \leq r < n$

- something about $H^n(K;\tilde{\omega}_n(Y))$ $\quad r>n$

Eilenberg-Machine