19.04.2021

Monday's Nonstandard Seminar 29

14:00

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Title: **Existence for a class of fourth-order quasilinear parabolic systems**

Abstract: Let us consider the system

$$(*) \qquad \boldsymbol{u}_t + \operatorname{div} \mathcal{B}(\boldsymbol{u}) \nabla \mathcal{A}(\boldsymbol{u}) = \operatorname{div} \boldsymbol{g},$$

where

- $\mathcal{A}(\boldsymbol{u}) = \operatorname{div} D\Phi(x, \nabla \boldsymbol{u})$,
- Φ is strictly convex, continuously differentiable and of p-growth with respect to the second variable,
- $\mathcal{B}(\boldsymbol{u}) = \mathbb{B}(t, x, \boldsymbol{u}, \nabla \boldsymbol{u}, \mathcal{A}(\boldsymbol{u}))$ with a Carathéodory, tensor-valued function $\mathbb{B}$ bounded away from 0 and ∞,
- $\boldsymbol{g} = \boldsymbol{g}(t, x)$ is square-integrable.

The system $(*)$ formally corresponds to the gradient flow of $\int_\Omega \Phi(\cdot, \nabla \boldsymbol{u})$ with respect to a weighted H^{-1} metric depending on the solution. However, in order to fit into the scope of known theory of gradient flows, we would need the metric to have some sort of time regularity, effectively restricting us to $\mathbb{B} = \mathbb{B}(t, x, \boldsymbol{u})$ with weakly differentiable dependence on t.

In the talk, I will discuss known existence results for fourth-order quasilinear parabolic systems and give a few examples of form $(*)$ that do not fit into the scope of existing theory. Then, I will sketch the proof of global-in-time existence of a weak solution to the initial value problem for $(*)$ in a spatially periodic domain. The proof is based on the energy inequality and a Galerkin approximation tailored to suit the special structure of the system, which allows us to deduce strong convergence of approximate $\nabla \boldsymbol{u}$ and $\mathcal{A}(\boldsymbol{u})$.

This is joint work with Yoshikazu Giga.