

1.03.2021

Monday's Nonstandard Seminar 22

14:00

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Title: **Maximal non-compactness of Sobolev embeddings**

Abstract: It has been known that sharp Sobolev embeddings into weak Lebesgue spaces are non-compact but the question of whether the measure of non-compactness of such an embedding equals to its operator norm constituted a well-known open problem. The existing theory suggested an argument that would possibly solve the problem should the target norms be disjointly superadditive, but the question of disjoint superadditivity of spaces $L^{p,\infty}$ has been open, too. In this paper, we solve both these problems. We first show that weak Lebesgue spaces are never disjointly superadditive, so the suggested technique is ruled out. But then we show that, perhaps somewhat surprisingly, the measure of non-compactness of a sharp Sobolev embedding coincides with the embedding norm nevertheless, at least as long as $p < \infty$. Finally, we show that if the target space is L^∞ (which formally is also a weak Lebesgue space with $p = \infty$), then the things are essentially different. To give a comprehensive answer including this case, too, we develop a new method based on a rather unexpected combinatorial argument and prove thereby a general principle, whose special case implies that the measure of non-compactness, in this case, is strictly less than its norm. We develop a technique that enables us to evaluate this measure of non-compactness exactly.

This is a joint work with Jan Lang (The Ohio State University, Columbus, OH), Vít Musil (Masaryk University, Brno) and Miroslav Olšák (University of Innsbruck).