

Author: Lars Diening (Bielefeld University)

Title: Elliptic Equations with Degenerate Weights

Abstract:

We study the regularity of elliptic equations with degenerate elliptic weights in the linear case

$$-\operatorname{div}(\mathbb{A}(x)\nabla u) = -\operatorname{div}(\mathbb{A}(x)G),$$

as well as in the non-linear case

$$-\operatorname{div}(|\mathbb{M}(x)\nabla u|^{p-2}\mathbb{M}^2(x)\nabla u) = -\operatorname{div}(|\mathbb{M}(x)G|^{p-2}\mathbb{M}^2(x)G),$$

where $1 < p < \infty$ and G is the given data. The mappings $\mathbb{A}, \mathbb{M} : \Omega \rightarrow \mathbb{R}_{\text{sym}}^{n \times n}$ are symmetric, positive definite, matrix-valued weights, which may be degenerate. This includes for examples simple weights as $|x|^{\pm\epsilon}\text{Id}$. We establish a novel condition on the weight \mathbb{M} . Instead of a BMO (bounded mean oscillation) smallness condition for \mathbb{M} , we use a BMO smallness condition on its logarithm $\log \mathbb{M}$, which is new even for the linear case. Under this condition we show that local higher integrability of G transfers to ∇u . The sharpness of our estimates is proved by examples.

The talk is based on joint work with Anna Balci, Raffaella Giova and Antonia Passarelli di Napoli.