29.03.2021

Monday's Nonstandard Seminar 26

14:00

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Title: **The Alexandrov Theorem in Minkowski spaces**

Abstract: Let $F : \mathbf{R}^n \to \mathbf{R}$ be a uniformly convex smooth norm. The *F-perimeter* of a Caccioppoli set $\Omega \subseteq \mathbf{R}^n$ is defined as

$$\mathscr{P}(\Omega) = \int_{\partial^* \Omega} F(\mathbf{n}(\Omega, x)) \, \mathrm{d}\mathscr{H}^{n-1}(x) \,,$$

where $\mathbf{n}(\Omega, x)$ denotes the Euclidean exterior unit normal of Ω at $x \in \partial^* \Omega$. We study critical points of $\mathscr{P}$ restricted to the family $\mathscr{A}$ of Caccioppoli sets $\Omega \subseteq \mathbf{R}^n$ with fixed volume $\mathscr{L}^n(\Omega) = 1$. Minima of $\mathscr{P}|\mathscr{A}$ solve the *anisotropic isoperimetric problem* and are well known, by the results of Jean Taylor from the 70's, to be *Wulff shapes*, i.e., balls with respect to the dual norm F^*. More recently He, Li, Ma, and Ge [Indiana Univ. Math. J., 2009] proved that critical points *with smooth boundaries* must be finite sums of Wulff shapes. Delgadino and Maggi [Anal. PDE, 2019] characterised critical points in case F is the Euclidean norm (in this case $\mathscr{P}(\Omega) = \mathscr{H}^{n-1}(\partial^* \Omega)$) but without any a priori assumptions on regularity of $\partial^* \Omega$.

I shall present my joint work with Antonio De Rosa and Mario Santilli [ARMA, 2020] in which we prove that critical points satisfying $\mathscr{H}^{n-1}(\partial \Omega \sim \partial^* \Omega) = 0$ must be finite sums of Wulff shapes.