Buttazzo ↓ Lavrentiev phenomenon

Przykład Manii

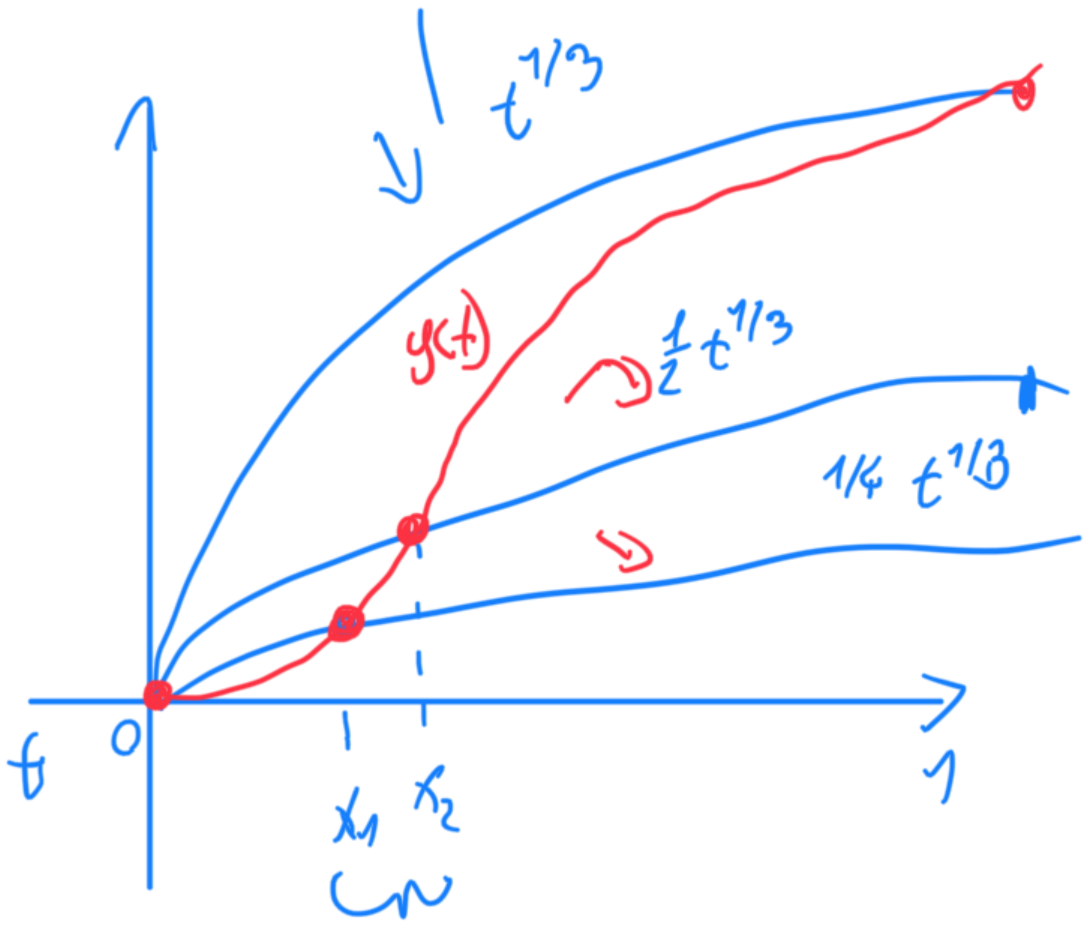
$$y \overset{F}{\longmapsto} \int_0^1 (y^3 - t)^2 (y'(t))^6 \, dt \qquad y(0)=0, \quad y(1)=1$$

uwaga: infimum po AC wynosi 0.

$$\inf_{y \in Lip[0,1]} F(y) = \eta > 0$$

$y(0)=0, \quad y(1)=1$

$$y \overset{F}{\longmapsto} \int_0^1 (y^3 - t)^2 (y'(t))^6 \, dt$$

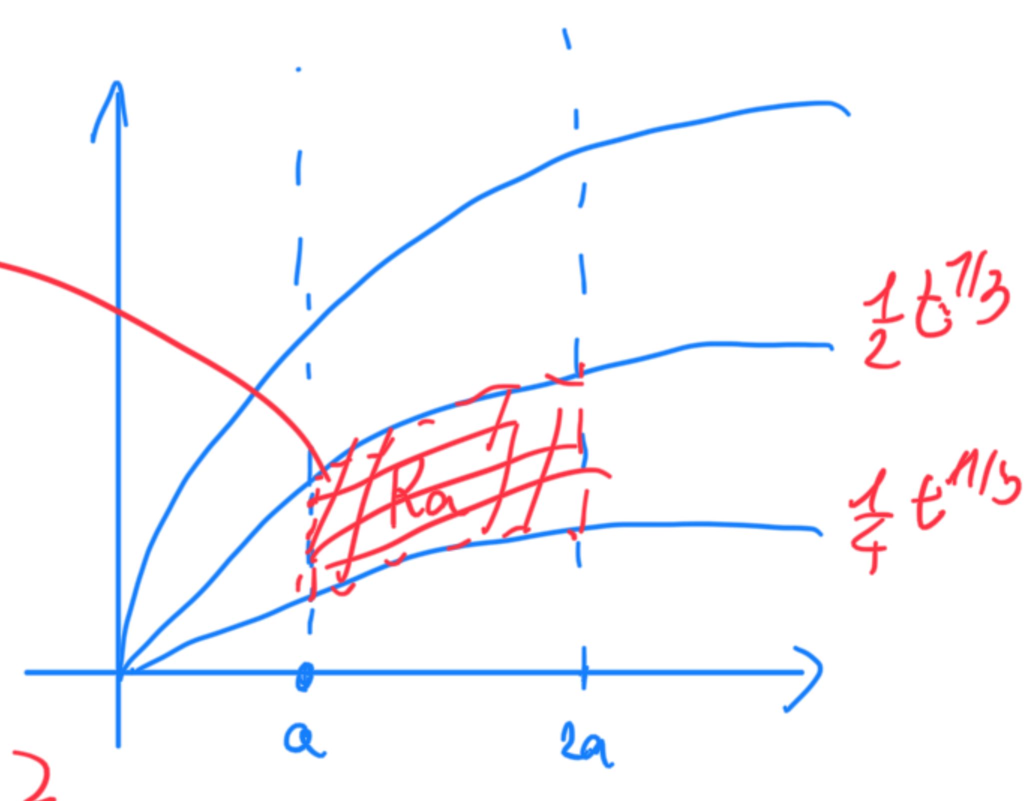


y jest Lip więc ma ograniczoną pochodną oraz jest w AC.

$\inf_{(y,t) \in R_a} (y^3 - t)^2$, wtedy $(\alpha^3 t - t)^2 = (\alpha^3 - 1)^2 t^2$

$y = \alpha t^{1/3}$, $\frac{1}{4} \le \alpha \le \frac{1}{2}$

$a \le t \le 2a$



$$\inf_{(y,t) \in R_a} (y^3 - t)^2 = \inf_{\substack{(\alpha,t) \\ \frac{1}{4} \le \alpha \le \frac{1}{2} \wedge \\ a \le t \le 2a}} (\alpha^3 - 1)^2 t^2 \ge \left(\frac{7}{8}\right)^2 a^2$$

$a \in (0, \frac{1}{2})$

$$\int (y^3(t) - t)^2 \, |y'(t)|^6 \, dt \ge \int^{x_2} (y^3(t) - t)^2 \, |y'(t)|^6 \, dt$$

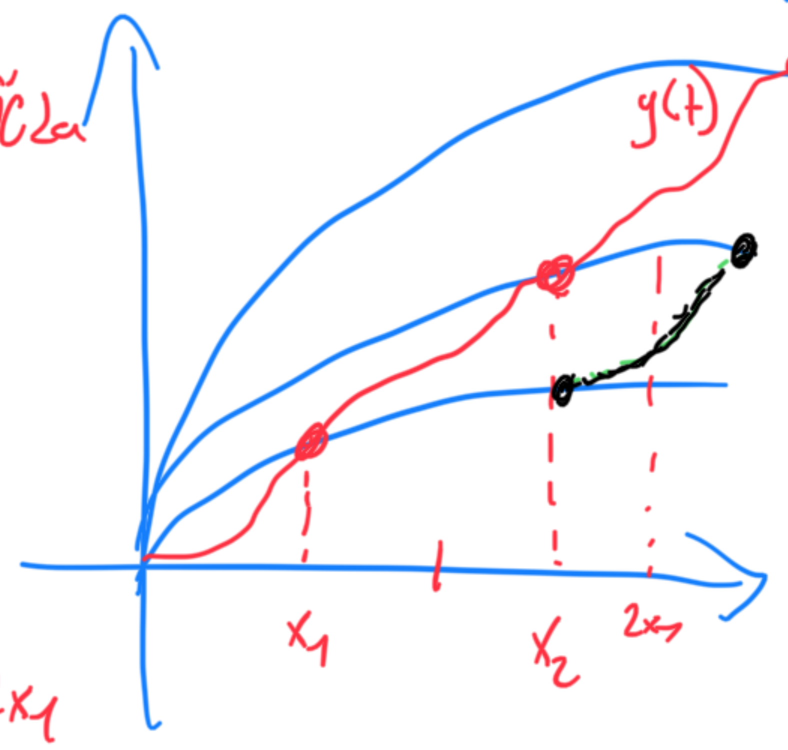
$\check{o}$ $\qquad\qquad$ x_1

$$\int_{x_1}^{x_2} |y'(t)|^6$$

$$\varphi\left(\frac{\int_a^b f}{b-a}\right) \le \frac{1}{b-a}\int_a^b \varphi\circ f \qquad \varphi(x) = |x|^6$$

$$\Rightarrow |x_1-x_2|^{-6}\, |y(x_2)-y(x_1)|^6 \le |x_1-x_2|^{-1} \int |y'(t)|^6$$

$a = x_1$, ale x_2 nie musi wynosić $2a$



1) $x_1 > \frac{1}{2}$

2) $x_1 < \frac{1}{2}$ $\nearrow$ $\begin{cases} x_2 \le 2x_1 \\ y(x_2) = \frac{1}{2}x_2^{1/3} \end{cases}$

$\searrow$ $x_2 \ge 2x_1$ ale można wziąć $x_2 = 2x_1$

$$\frac{1}{4}x_2^{1/3} \le y(x_2) \le \frac{1}{2}x_2^{1/3}$$

1) $x_1 > \frac{1}{2}$

$$\int_{x_1}^{x_2} (y^3-t)^2\, |y'(t)|^6 \ge \left(\frac{7}{8}\right)^2\left(\frac{1}{2}\right)^2 \int_{x_1}^{x_2} |y'(t)|^6\, dt$$

stała

$$\ge k\,|x_1-x_2|^{-5}\, |y(x_1)-y(x_2)|^6$$

$$y(x_1) = \frac{1}{4}x_1^{1/3}$$

$$y(x_2) = \frac{1}{2}x_2^{1/3}$$

$$\frac{1}{4}x_1^{1/3} = \frac{1}{2}x_2^{1/3}$$

$$x_1 = 8x_2,$$

zeruje się dla $\left|\frac{1}{4}x_1^{1/3} - \frac{1}{2}x_2^{1/3}\right|^6$

$$\ge \left|\frac{1}{4}x_1^{1/3} - \frac{1}{2}x_2^{1/3}\right|$$

[illegible]

$\left|\frac{1}{4} x_1^{\ldots}\right| = x_1 \cdot \left(\frac{1}{4}\right)$

Przypadek $x_1 < \frac{1}{2}$

$x_2 \leq 2x_1$ oraz $y(x_2) = \frac{1}{2} x_2^{1/3}$

Musimy oszacować dostajemy $|x_1 - x_2|^{-5} \left|\frac{1}{4} x_1^{1/3} - \frac{1}{2} x_2^{1/3}\right|^6 x_1^2$

$\Rightarrow \int_{x_1}^{x_2} (y^3(t) - t)^2 |y'(t)|^6$

ale wewnątrz obszaru mamy

$\underbrace{(y^3(t) - t)^2} \geq \left(\frac{7}{8}\right)^2 x_1^2$

$|x_1 - x_2| \leq x_1$

$|x_1 - x_2|^{-5} \geq x_1^{-5}$

$\underbrace{\left|\frac{1}{4} x_1^{1/3} - \frac{1}{2} x_2^{1/3}\right|^6} \geq x_1^2 \cdot C$

x_1^2

$$\int_{x_1}^{x_2} (y^3(t) - t)^2 |y'(t)|^6 \geq x_1^{-1} \cdot K, \text{ oraz}$$

$$\boxed{\frac{1}{x_1} \cdot K}, \quad 0 \leq x_1 \leq \frac{1}{2}$$

Przypadek 3

$x_1 < \frac{1}{2} \quad y(x_2) \leq \frac{1}{2} x_2^{1/3}$

$x_2 = 2x_1$

$$\left|\frac{1}{4} x_1^{1/3} - y(x_2)\right|^6$$

$$\geq \left|\frac{1}{4} x_1^{1/3} - \frac{1}{4} (2x_1)^{1/3}\right|^6$$

$$\geq (\ldots) x_1^2$$