Infinite Automata 2025/26

Starred Exercises Series 1

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Submit solutions by emailing them to Henry. Please include "Infinite Automata" in the subject line.

Star Exercise 1.1. Let $V = (\Sigma, Q, T, q_0, F)$ be a letter-labelled 2-VASS with the coverability acceptance condition. Prove or disprove: the Parikh image of the language recognised by V is semilinear. Note: a VASS with the coverability acceptance condition accepts a word w if there is a run that reads w and that ends at an accepting state $q \in F$ with *any* counter values $\mathbf{v} \geq 0$.

Star Exercise 1.2. Let $V = (Q, T)$ be a d-VASS. We call a configuration $(p, \mathbf{u})$ *pumpable in V* if there exists a configuration $(p, \mathbf{v})$ such that $(p, \mathbf{u}) \xrightarrow{*}_V (p, \mathbf{v})$, $\mathbf{v} \geq \mathbf{u}$, and $\mathbf{v} \neq \mathbf{u}$. Prove or disprove: if there is a configuration that is pumpable in V, then there is a configuration $(p, \mathbf{u})$ that is pumpable in V such that $\|\mathbf{u}\|_\infty \leq \mathcal{O}(2^{n^C})$ where n is the size of V encoded in binary and $C \in \mathbb{N}$ is some constant.

Star Exercise 1.3. Fix an alphabet Σ. Prove or disprove: for every $d \in \mathbb{N}$, there exists a letter-labelled d-VASS V_d with the coverability acceptance condition such that there does not exist a letter-labelled $(d-1)$-VASS V' with the coverability acceptance condition that recognises the same language.

Star Exercise 1.4. Show that reachability in binary-encoded 3-VAS is PSPACE-hard. Note: the 3-VAS $V \subseteq \mathbb{Z}^3$, the initial configuration $\mathbf{u} \in \mathbb{N}^3$, and the target configuration $\mathbf{v} \in \mathbb{N}^3$ are all encoded in binary.

Star Exercise 1.5. Show that for every $k \in \mathbb{N}$, there exists a binary 2-VAS V_k, an initial configuration $\mathbf{s}_k \in \mathbb{N}^2$, and a target configuration $\mathbf{t}_k \in \mathbb{N}^2$ such the the following conditions hold.

(i) V_k contains at least k transitions (i.e. $|V_k| \geq k$).

(ii) There is exactly one run from $\mathbf{s}_k$ to $\mathbf{t}_k$ in V_k.

(iii) The run from $\mathbf{s}_k$ to $\mathbf{t}_k$ uses every transition in V_k at least once.