Infinite Automata 2025/26

Exercise Sheet 2

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Exercise 2.1. Prove that reachability binary-encoded 1-VASS is NP-hard.

Hint. Reduce from the subset sum problem.

Exercise 2.2. Prove that reachability binary-encoded 1-VAS (where there are no control states) is NP-hard.

Exercise 2.3. Prove that, when at least one transition is positive and at least one transition is negative, the reachability problem in binary-encoded 1-VAS is in P.

Exercise 2.4. Prove that reachability binary-encoded 1-VASS is in PSPACE.

Hint. Use Lemma 1.10 and ideas in the proof of Theorem 1.8 (from Lecture 1).

Exercise 2.5. Consider a path in a finite automaton with n states. Let its skeleton be produced by repeating following procedure. Traverse the path and once a simple cycle is observed, we remove the simple cycle from the path if it only contains states which have already been visited *before* the cycle. Show that the length of the skeleton is $\mathcal{O}(n^2)$.

Exercise 2.6. Prove that if the following statement is assumed to be true, then the reachability problem in 1-VASS is in 2-EXPSPACE (doubly-exponential space). If there is a run from configuration c to configuration c' in a given 1-VASS V of size n, then there is run from c to c' of length $\ell \leq \mathcal{O}(2^{2^{2^n}})$. Now assume that there exists such a run with $\ell \leq \mathcal{O}(2^n)$; prove that reachability is in PSPACE.

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