GAeL XXI

Stockholm, 2013

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00-9:30	**Registration**				
9:30-10:00	Szemberg	Szemberg	Lazarsfeld	Kuznetsov	Lazarsfeld
10:00-10:30					
10:40-11:10	Garcia	Hawes	GAeL XXII	Gudim	Meachan
11:10-11:30	**Coffee break**				
11:30-12:00	Lazarsfeld	Lazarsfeld	Kuznetsov	Szemberg	Szemberg
12:00-12:30					
12:30-14:30	**Lunch and free discussion time**				
14:30-15:00	Kuznetsov	Kuznetsov	Free afternoon	Di Natale	Tanturri
15:00-15:30				Lesieutre	Lozovanu
15:30-16:00	**Coffee break**			**Coffee break**	
16:00-16:30	Blechschmidt	Litt		Sanna	Huizenga
16:40-17:10	Battiston	Greiner		Michałek	
Evening	**Poster session**	**Conference dinner**			

Mini-courses

- **Alexander Kuznetsov**

  Title: Derived Categories of Coherent Sheaves

  Abstract: Derived category of coherent sheaves is a convenient environment of investigating algebraic geometry of a variety. It provides useful techniques and gives a perspective point of view. On the level of derived categories one can see unexpected connections which are not visible on the classical level. I will try to give an introduction into the techniques of derived categories and semiorthogonal decomposition and (hopefully) will present many examples of applications of derived categories in algebraic geometry.

- **Robert Lazarsfeld**

  Title: Regularity and Syzygies of Algebraic Varieties

  Abstract: Over the past thirty or so years, classical work concerning the equations defining projective varieties has been generalized in two directions. First, there has been a great deal of interest in bounding the Castelnuovo-Mumford regularity of a variety, which controls its overall algebraic complexity. A second line of investigation centers on more refined algebraic invariants involving syzygies, and their relation to the geometry of the variety. I will give an introduction to this circle of ideas, with an emphasis on some of the many open problems in the area.

- **Tomasz Szemberg**

  Title: Seshadri Constants and Nagata Conjectures

  Abstract:

  1. General definitions and properties. Known bounds, open problems.

2. Specific examples, in particular abelian varieties.
3. Multi-point Seshadri constants, relations to Nagata-type conjectures.
4. Nagata-type conjectures for higher dimensional linear subspaces.

Junior talks

- **Giulia Battiston**

  Title: $\mathcal{D}$-modules in families in positive characteristic.

  Abstract: On smooth varieties X over algebraically closed fields k of positive characteristic the notion of $\mathcal{D}$-module, i.e. a coherent $\mathcal{O}_X$ module endowed with an action of the sheaf of differential operators, has interesting parallelisms with the characteristic zero case, for example it allows to define the monodromy group of a $\mathcal{D}$-module. In this talk I will consider a smooth family of $\mathcal{D}$-modules on $X \to S$ and look at the following question: if on every closed fiber the $\mathcal{D}$-module is trivial on some tale finite cover, what can I deduce on the geometric generic fiber? In the characteristic zero case Y. Andr has proved that also on the generic geometric fiber the bundle is trivialized by an tale finite cover. In positive characteristic H. Esnault and A. Langer proved the same for projective families, with some additional conditions. If the projective condition drops, this is not always the case, I will explain in which cases and try to give an idea on why this difference between zero and positive characteristic arises.

- **Ingo Blechschmidt**

  Title: Using the internal language of topoi in algebraic geometry

  Abstract: There are several important topoi associated to a scheme, for instance the petit and gros Zariski topoi. These come with an internal mathematical language which closely resembles the usual formal language of mathematics, but is "local on the base scheme":

  For example, from the internal perspective, the structure sheaf looks like an ordinary local ring (instead of a sheaf of rings with local stalks) and vector bundles look like ordinary free modules (instead of sheaves of modules satisfying a certain condition). The translation of internal statements and proofs is facilitated by an easy mechanical procedure.

  The talk will give an introduction to this topic and show how the internal point of view can be exploited to give simpler definitions and more conceptual proofs of the basic notions and observations in algebraic geometry. We will also point out certain unexpected properties of the internal universe, for instance that the structure sheaf looks in fact even like a field.

- **Carmelo Di Natale**

  Title: An Overview of Derived Deformation Functors

  Abstract: While Deformation Theory deals with functors of Artin rings, Derived Deformation Theory is about some sort of homotopy version of these, i.e. functors defined on the category of dg Artin algebras and satisfying suitable geometrical assumptions. One of the main reasons to move from the classical setting to the derived one is that here we obtain a full description of obstructions and cotangent complex. In my talk I will explain various approaches to Derived Deformation Theory (Kontsevich, Hinich, Manetti) and the way they have recently been unified (Pridham). Time permitting, I will conclude by mentioning some recent work about the interpretation of Griffiths' period map, which connects deformations of a smooth complex variety to those of its Hodge structures, in terms of Derived Deformation Theory.

- **Miguel Angel Garcia**

  Title: Grothendieck Duality

  Abstract: Let's consider a separated morphism $f : X \to Y$ between two algebraic varieties. We shall briefly discuss how homotopy theory in triangulated categories (in particular, Brown's representability theorem) provides us the existence of a right adjoint of the direct image $Rf_* : D(X) \to D(Y)$, which we denote $f^!$. From the adjunction formula we shall see that the difficulty of computing this $f^!$ relies on knowing the dualizing complex of the morphism considered, namely $f^!(\mathcal{O}_Y)$.

  Secondly we shall introduce the graphic method, which allows us to simplify the problem above to the case of a finite morphism (the embedding in the graphic).

  In the rest of the talk we shall discuss how this dualizing complex can be easily computed in the case of a Gorenstein or Cohen-Macauly morphism, thanks to the graphic method. We shall deduce well-known results such as Serre and Poincare duality.

- **Ulrike Greiner**

  Title: On the Chow ring of birational irreducible symplectic varieties

  Abstract: An irreducible symplectic variety (also known as hyperkhler manifold) over $\mathbb{C}$ is a simply connected, nonsingular, complex projective variety with a nowhere degenerated two-form σ generating $H^0(X, \Omega^2_X)$. Two important series of examples are provided by Hilbert schemes of points $\mathrm{Hilb}^n(S)$ of K3 surfaces and generalized Kummer varieties $K_n(A)$ of abelian surfaces.

  Consider two birational irreducible symplectic varieties X and X' . Huybrechts showed that they can be deformed in families $\mathcal{X}$ and $\mathcal{X}'$ of not necessarily projective hyperkhler manifolds over a one-dimensional manifold, which are isomorphic away from the special fibre. Under certain conditions on X and X', these families exist indeed as algebraic varieties. In particular, this holds if X and X' are isomorphic in codimension two or if they are connected by a general Mukai flop (if one of these conditions holds, we will say that X and X' satisfy (†)). The central result which I will present is the following:

  Theorem 1.1. Let X and X' be birational irreducible symplectic varieties satisfying (†). Then there exists a correspondence $[Z]_* : CH(X) \xrightarrow{\simeq} CH(X')$ which is an isomorphism of graded rings.

  Even in very special cases (e.g. for elementary Mukai flops) this result has not been known before. After introducing all necessary notation and presenting the theorem, I will specify what $[Z]_*$ is, sketch the proof of the theorem and present a first application: There are conjectures of Beauville and Voisin, predicting injectivity of the cycle class map on certain subalgebras of the Chow ring. Beauville proves the invariance of his conjecture under elementary Mukai flops. By means of Theorem 1.1, we extend this result to birational correspondences satisfying (†) and to the stronger conjecture of Voisin.

- **Mikhail Gudim**

  Title: On cohomology of Kempf-complexes

  Abstract: In the appendix of the famous book Commutative Algebra with a View Towards Algebraic Geometry one can find an infinite family of complexes indexed by integers. This family includes Eagon-Northcott and Buschsbaum-Rim complexes. The objective of this paper is to study this family, and, in particular, refine the knowledge of its cohomology.

  First, we obtain these complexes from the derived images of twists of the Koszul complex on the projective space. This idea apparently goes back to Kempf [1970]. To acknowledge it, we refer to this family as Kempf complexes. Taking this geometric point of view, we interpret the cohomology

of these complexes as the cohomology of certain vector bundles on the projective space, and proceed with calculations.

Finally, we put the above results in the realm of tilting theory: nonexactness of this family in certain regions can be seen as a failure of the exceptional sequence of line bundles on the projective space to lift to an exceptional sequence on a certain vector bundle. This observation creates a curious contrast with the results of Buchweitz-Leushke-Van den Bergh, stating that the exceptional sequence of twisted differential forms does lift to the same vector bundle.

- **Thomas Hawes**

  Title: Non-reductive Geometric Invariant Theory

  Abstract: We will give an outline of Geometric invariant theory (GIT) for non-reductive group actions on projective varieties. When a reductive group G acts on a projective variety X, Mumford showed how to find an open subset X^s of X of 'stable points' (depending on a linearisation of the action) that admits an orbit space variety X^s/G, and this quotient admits a canonical compactification $X//G$. This rather nice picture breaks down when the group G is not reductive, due to the possibility of non-finitely generated rings of invariants. This talk will look at joint work with Brent Doran and Frances Kirwan, describing how one can still find sets of stable points that admit orbit spaces and construct completions of these, which are canonical when the ring of invariants is finitely generated.

- **Jack Huizenga**

  Title: Higher rank interpolation problems and the birational geometry of Hilbert schemes of points

  Abstract: Questions like the Nagata conjecture seek to determine when certain zero-dimensional schemes impose independent conditions on sections of a line bundle on a surface. Understanding analogous questions for vector bundles instead of line bundles amounts to studying the birational geometry of Hilbert schemes of points on a surface. In this talk we will discuss how derived categories of sheaves and the notion of Bridgeland stability allow one to solve interesting higher rank interpolation problems. We compute the cone of effective divisors on the Hilbert scheme of points in the plane and study the stable base locus decomposition of this cone.

- **John Lesieutre**

  Title: A divisor with non-closed diminished base locus

  Abstract: I will explain the construction of a pseudoeffective $\mathbb{R}$-divisor D on the blow-up of $\mathbb{P}^3$ at nine very general points which has negative intersections with a Zariski dense set of curves. The diminished base locus $\mathbf{B}_-(D) = \bigcup_{A \text{ ample}} \mathbf{B}(D + A)$ of D is not closed, and D does not admit a Zariski decomposition in even a very weak sense. By a similar method, I'll exhibit an $\mathbb{R}$-divisor which is nef on very general fibers of a family, but fails to be nef over countably many prime divisors in the base. I'll also discuss some related issues for divisors on Calabi-Yau threefolds.

- **Daniel Litt**

  Title: Motivic Analytic Number Theory

  Abstract: There are beautiful and unexpected connections between algebraic topology, number theory, and algebraic geometry, arising from the study of the configuration space of points on a variety. I'll discuss several ideas and open conjectures surrounding these connections, and describe the proof of one of these conjectures (relating the Hodge theory of a variety to the geometry of points on the variety) in the case of curves and algebraic surfaces. Everything in the talk will be defined from scratch, and should be quite accessible.

- **Victor Lozovanu**

  Title: An extension of Kodaira vanishing in arbitrary codimension

  Abstract: One of the most celebrated theorems in complex algebraic geometry is Kodaira vanishing, together with its extension due to Kawamata and Viehweg. In this talk I will discuss about a joint work with Greg G. Smith, where we generalize Kodaira vanishing to arbitrary codimension. I will present a few applications to this work; by giving answers to questions of projective normality and bounding the multigraded regularity. I will also discuss some future projects that derive nicely from this work.

- **Ciaran Meachan**

  Title: Derived autoequivalences of hyperkahler varieties

  Abstract: $\mathbb{P}^n$ functors are a natural generalisation of Huybrechts and Thomas' $\mathbb{P}^n$ objects and as such, they determine autoequivalences of the codomain category. We will report on the latest developments in this direction when the codomain category is the derived category of coherent sheaves on a hyperkahler variety.

- **Mateusz Michałek**

  Title: Generators of toric ideals

  Abstract: Toric ideals are objects appearing on the intersection of algebra, algebraic geometry and combinatorics. We present some of the open problems concerning the study of generators of toric ideals. We show new results on projective schemes defined by homogeneous toric ideals. Our method was successfully applied in such topics as group-based models in phylogenetics and White's conjecture in matroid theory.

- **Giangiacomo Sanna**

  Title: Instantons on Fano 3-folds

  Abstract: Instantons are a class of self-dual vector bundles on $\mathbb{P}^3$. Their moduli space has been investigated via monads and restrictions to lines (Grauert-Mulich-type theorems). In this talk I will introduce a generalization of instantons to other Fano 3-folds, and motivate it from the derived categorical viewpoint. I will then describe some attempts to recover the monadic and the Grauert-Mulich description in this new setting and discuss the relation with nets of quadrics.

- **Fabio Tanturri**

  Title: Hilbert schemes of degeneracy loci

  Abstract: Given a morphism F between vector bundles on $\mathbb{P}^N = \mathbb{P}(V)$, its (first) degeneracy locus X is made up of points in which the morphism has not maximal rank; locally, X is cut by the maximal minors of the matrix representing F.

  Interesting questions arise when one wants to give a parametrization of such degeneracy loci, by looking at the union of the irreducible components of the Hilbert Scheme H_X containing the general X's. I will focus on the case in which the morphism is given by m global sections of $\Omega(2)$, the twisted cotangent sheaf on $\mathbb{P}(V)$. In this case, making use of the Kempf-Weyman's method for computing syzygies via resolutions of singularities, H_X can be proved to be birational to the Grassmannian $\mathrm{Gr}(m, \Lambda^2(V))$, when $3 < m < N + 1$. If $m = 3$, the Grassmannian is birational to a subscheme of H_X, whose complement can be characterized geometrically.