CURVES IN $C^{(2)}$

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Introduction

LET S be a surface of general type with irregularity $q > 0$ and maximal Albanese dimension. In [2] it is proved that if there exists a curve $C \subset S$ with $p_a(C) = q(S)$, 2-connected, with $C^2 > 0$ then S is birational to the symmetric square of a curve. In fact, for C smooth and irreducible and S minimal, S is isomorphic to $C^{(2)}$.

In [1] the same authors ask whether there is an example of a curve C contained in an irregular surface S with $C^2 > 0$ and $q(S) < p_a(C) < 2q(S) - 1$. We would like to see wether such a curve exists in the symmetric square of a curve. If a curve C in an irregular surface S has $g(C) < 2q(S) - 1$ then, by a theorem of Xiao $h^0(\mathcal{O}_S(C)) = 1$, that is, C does not move linearly. It is known that

Lemma 1. *Let $B \subset C^{(n)}$ be a curve such that:*

i) $B \not\subset Supp(C_p)$ $\forall p \in C$ (where C_p denotes the coordinate divisor parametrising all degree n divisors in C through $p \in C$)

ii) $B \cdot C_p = 1$

Then B is smooth and there exists a degree n covering $f : C \to B$. In particular, $\Sigma = \{f^{-1}(q) : q \in B\} \subset C^{(n)}$ is smooth and isomorphic to B.

In particular, every curve which has C as degree n covering lies in $C^{(n)}$.

Definition 1. *We say that a diagram of morphisms of curves*

$$\begin{array}{ccc} \Gamma & \xrightarrow{(n:1)} & B \\ {\scriptstyle (d:1)}\downarrow & & \\ C & & \end{array}$$

***completes** if there existst a curve D such that we can complete the previous diagram to obtain a commutative one*

$$\begin{array}{ccc} \Gamma & \xrightarrow{(n:1)} & B \\ {\scriptstyle (d:1)}\downarrow & & \downarrow{\scriptstyle (d:1)} \\ C & \xrightarrow[(n:1)]{} & D \end{array}$$

My research

AMONG my research interests there are the study of curves lying in the symmetric square of a curve, and the study of the existence of curves with low genus and positive self-intersection.

The first observation is that if $\tilde{B}$ is an irreducible curve in $C^{(2)}$ not contained in the diagonal, and B is not covered by C then, by lemma 1 and because B is not covered by C and C_p is ample in $C^{(2)}$, we have $\tilde{B} \cdot C_p = d \geq 2$. We have proven that

Theorem 1. *Let d be a prime number. Given an irreducible curve $\overline{B}$, with normalization B and $g(C) < g(B)$, a birrational model $\tilde{B}$ lies in $C^{(2)}$ with $d = \tilde{B} \cdot C_p$ if, and only if, there exists a diagram*

$$\begin{array}{ccc} \Gamma & \xrightarrow{(2:1)} & B \\ {\scriptstyle (d:1)}\downarrow & & \\ C & & \end{array}$$

which does not complete, where Γ is a smooth irreducible curve.

As we are interested in curves with positive self-intersection, we have computed $\tilde{B}^2$ for the image of B, $\tilde{B}$, in $C^{(2)}$

$$\tilde{B}^2 = g(\Gamma) - 1 - d(2g(C) - 2) + 2(p_a(\tilde{B}) - g(B)) + \sigma,$$

where σ depends on the intersection between the graphs of the two morphisms in $\Gamma^{(2)}$.

To construct examples of curves in symmetric squares we use the action of non commutative groups in curves and we consider the quotients by appropiate elements. Looking, for instance, hyperelliptic curves and their automorphism grups.

References

[1] M. Mendes Lopes, R. Pardini, and G. P. Pirola, *Brill-Noether loci for divisors on irregular varieties*, ArXiv e-prints (2011).

[2] M. Mendes Lopes, R. Pardini, and G. P. Pirola, *A characterization of the symmetric square of a curve*, International Mathematics Research Notices (2011).