Linear systems containing the general curve of genus g

Let $\overline{M}_g$ and $\overline{M}_{g,n}$ be respectively the (coarse) moduli space of stable and n-pointed stable complex curves of genus $g \geq 3$.
It is a well-known fact that the uniruledness of $\overline{M}_g$ is equivalent to the existence of a non-ruled surface S carrying a positive-dimensional linear system containing the general curve of genus g.
It will be shown how to use linear systems of this kind to construct rational curves on some $\overline{M}_{g,n}$ passing through the general point, hence proving their uniruledness. The proof uses Brill-Noether theory and characteristic linear series.
Using similar arguments it will be shown how to prove uniruledness for some pointed hyperelliptic loci.
In the end some geometric conditions for the existence of linear systems as above will be examined. Conditions on the geometric genus of S and on the slope of "induced" fibrations will be given.