

XX GAeL

June 18-22, Grenoble

<http://www.mimuw.edu.pl/~gael/>

---

# Introduction to derived algebraic geometry

Bertrand Toën

Université de Montpellier 2

These lectures are an introduction to the notion of derived schemes and derived stacks, the main objective being to construct various derived moduli stacks associated to natural moduli problems (mainly sheaves on projective manifolds, but not only).

## **Lecture 1: Motivations**

Moduli of finite dimensional modules as a motivating example. Non-representability and non-naturality of the obstruction theory.

## **Lecture 2: Local derived algebraic geometry**

Simplicial commutative algebras and their homotopy theory, the cotangent complex, smooth, étale and flat morphisms.

## **Lecture 3: Derived schemes and stacks**

The category of derived stacks, definitions of derived schemes and derived Artin stacks. Relations with the categories of schemes and Artin stacks. Derived fiber products and derived mapping spaces.

## **Lecture 4: Representability of derived moduli problems**

Finite dimensional modules, vector bundles and coherent sheaves on projective varieties. The derived stack of stable maps. Virtual structure sheaves and virtual classes. Existence of symplectic structures.

---