XX GAeL

June 18-22, Grenoble

`http://www.mimuw.edu.pl/~gael/`

Moduli of K3 surfaces and irreducible symplectic manifolds

Klaus Hulek

Leibniz Universität Hannover

Lecture 1: K3 Surfaces and irreducible holomorphic symplectic manifolds
We shall introduce K3 surfaces and irreducible holomorphic symplectic manifolds (IHSM) and discuss their most important properties. In particular we shall define the K3 lattice and the Beauville lattice for IHSM and use these to define the period map.

Lecture 2: The Torelli theorem
The Torelli theorem for K3 surfaces says that one can recover a (polarized) K3 surface from its period point. This can be used to construct moduli spaces of polarized K3 surfaces. The corresponding result for IHSM is considerably more subtle and we shall discuss recent work by Verbitsky, Markman and Huybrechts which also allows one to construct moduli spaces of polarized IHSM as periods of a type IV domain by an arithmetic group.

Lecture 3: Automorphic forms and their relevance to the geometry of moduli spaces
Here we shall introduce the concept of automorphic forms and their relationship with pluricanonical forms on modular varieties. We shall discuss under which conditions these forms can be extended to a smooth projective model of a modular variety and how this can be used to prove results about the Kodaira dimension of the moduli spaced of polarized K3 surfaces or of polarized IHSM.

Lecture 4: Quasi-pullbacks of the Borcherds form
In order to prove that a given modular variety has general type one often wants

to construct low weight cusp forms which vanish on certain Heegener divisors. Such forms can sometimes be constructed as the quasi-pullback of Borcherds' modular form of weight 12 which lives on a 26-dimensional modular variety. We shall explain the construction of such quasi-pullbacks. This will lead us naturally into questions concerning the geometry of root lattices and analytic number theory.