# Problem Sheet on K3 surfaces and IHSM

Lecture 3

(1) Let $\mathcal{H}_n$ be the tubular domain associated to a lattice $L = 2U \oplus L_0$ as in lecture 3 and consider the action of $g \in \mathrm{O}(L)$ on $\mathcal{H}_n$. Calculate the determinant of the functional matrix (thus verifying that it is $\det(g)j(g, Z)$).

(2) Let $\tilde{\omega} = d\tilde{z}_1 \wedge \ldots \wedge \tilde{z}_n$ and let $F \in M_{nk}(\mathrm{O}(L), \det^{\otimes k})$. Show that $\tilde{F}\tilde{\omega}^k$ is an $\mathrm{O}(L)$-invariant pluricanonical form.

(3) Show that every non-constant modular form vanishes on a non-empty divisor (if $n \geq 2$) (Koecher principle).

(4) Show the following: If $(S, h)$ is a semi-polarized K3 surface of degree $2d$ which contains a smooth rational curve $R$ with $R.h = 0$, then the period point of $(S, h)$ lies on a reflection hyperplane.

(5) Construct examples of polarized K3 surfaces $(S, h)$ of degree $2d$ such that the point $[(S, h)] \in \mathcal{F}_{2d}$ is singular.

(6) Show that for every prime $p$ there is a finite surjective map $\mathcal{F}_{2p^2} \to \mathcal{F}_2$.

(7) For which values of $d$ can you show that $\mathcal{F}_{2d}$ is unirational?