Problem Sheet on K3 surfaces and IHSM

Lecture 1

(1) Prove that the following are K3 *surfaces*: double covers of $\mathbb{P}^2$ branched along a sextic, quartics in $\mathbb{P}^3$, complete intersections of bidegree $(2,3)$ in $\mathbb{P}^5$ or of multidegree $(2,2,2)$ in $\mathbb{P}^6$. Can you think of other examples which are complete intersections?

(2) Analyze what happens when you take double covers of $\mathbb{P}^2$ branched along a sextic with "mild" singularities.

(3) Count the number of moduli in the above examples.

(4) Find K3 surfaces with high Picard number (Hint: look e.g. at quartics in $\mathbb{P}^3$ with many automorphisms).

(5) Show that the Kummer surface, i.e. the minimal resolution of the quotient A/ι of an abelian surface A by the invoultion $\iota : x \mapsto -x$, is a K3 surface. What can one say about the Picard group?

(6) The degree Hilbert scheme of a K3 surfaces S can be constructed as the blow-up of the second symmetric product $S^2(S) = S \times S/S_2$ along the diagonal. Prove that there exists a nowhere vanishing holomorphic symplectic 2-form on $\mathrm{Hilb}^2(S)$.

(7) Prove that the generalized Kummer variety $K^n(A)$ is an IHSM.