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Junior talks

Pawel Borowka
Curves on $(1,3)$ polarised abelian surfaces

There is a well known theory of symmetric theta divisors on principally polarised abelian surfaces. The less known fact is that we can perform a similar construction on a $(1,3)$ surface. I will discuss a few properties of curves obtained in this way.

Salvatore Cacciola
On the semiampleness of the positive part of Zariski decompositions

In the context of the minimal model program, it is usual to work with pairs (X,Δ), where X is a projective normal variety and Δ is a $\mathbb{Q}$-divisor on X. An important theorem of Kawamata says that, on pairs of (log) general type of any dimension, the positive part of the Zariski decomposition of the log canonical divisor $K_X+\Delta$, provided it exists, has a base point free multiple if the pair has resonably good singularities (KLT). The same holds if we consider Zariski decompositions of big divisors whose difference with the log canonical is nef. After reviewing some standard notions about singularities of pairs, I will present possible generalizations of the latter result to the more general case of LC pairs. The main issue is the behaviour of our divisor at the so called Non-klt locus, the closed subset where the pair fails to be KLT.

Noah Giansiracusa
Conformal Blocks and Rational Normal Curves

I'll discuss a result that the Chow quotient parametrizing configurations of n points in P^d which generically lie on a rational normal curve is isomorphic to $M_{0,n}$, generalizing the well-known $d=1$ result of Kapranov. The corresponding GIT quotients, for symmetric linearization, are related to certain line bundles coming from the genus zero WZW model in conformal field theory. A representation-theoretic symmetry is manifest as the classical Gale transform in this setting.

Christopher Allen Manon
Conformal blocks and presentations of projective coordinate rings for the

moduli of principal parabolic $SL_2(\mathbb{C})$ bundles on a punctured projective curve

For G a simple Lie group, the moduli $\mathcal{M}_{C,\vec{p}}(G)$ of principal G bundles on a complex projective curve C with parabolic structure at the points $(p_1, \ldots, p_n) = \vec{p}$ carries a line bundle $\mathcal{L}(\vec{\lambda}, L)$ for every n-tuple of dominant G weights $\vec{\lambda}$ and non-negative integer L. Besides being of interest in the context of the moduli problem, the space of global sections $H^0(\mathcal{M}_{C,\vec{p}}(G), \mathcal{L}(\vec{\lambda}, L))$ of this line bundle appears in conformal field theory as a space of conformal blocks on C of level L for the conformal field theory defined by the Kac-Moody algebra $\hat{\mathfrak{g}}$ attached to the Lie algebra of G. These spaces also carry information on the representation theory of G, as they are isomorphic to the space of G-invariant vectors in the tensor product $V(\lambda_1) \otimes \ldots \otimes V(\lambda_n)$ of irreducible G-representations when L is large.

We consider these spaces in an algebraic context by studying the projective coordinate ring $R_{\vec{\lambda},L} = \bigoplus_{N=0}^{\infty} H^0(\mathcal{M}_{C,\vec{p}}(G), \mathcal{L}(N\vec{\lambda}, NL))$ of $\mathcal{L}(\vec{\lambda}, L)$. We will explain how the fusion rules from conformal field theory endow this algebra with a combinatorial multiplication rule based on graphs labeled by dominant weights of G. In general this leads to a flat deformation of $R_{\vec{\lambda},L}$ which simplifies the structure of this algebra and gives a toric deformation in the case $G = SL_2(\mathbb{C})$. We will report on recent progress on the commutative algebra of $R_{\vec{\lambda},L}$ obtained using these methods, including a structural description of the polytopes obtained in the $SL_2(\mathbb{C})$ case along with classification results on generators and relations.

Steffen Marcus

Polynomial Families of Tautological Classes on the Moduli Space of Curves

The tautological ring is a heavily studied subring of the Chow ring of the moduli space of curves. In this talk, I will describe natural families of tautological classes, called double Hurwitz classes, which arise by pushing forward the virtual fundamental classes of spaces of relative stable maps to an unparameterized projective line. "Relative" in this case means our maps have prescribed ramification over zero and infinity given by partitions of the degree. We compare these to classes arising from sections of the universal Jacobian which are polynomial in the parts of the partitions indexing the special ramification data.

This is joint work with Renzo Cavalieri and Jonathan Wise.

Clélia Pech

Quantum cohomology of the odd symplectic Grassmannian

Odd symplectic Grassmannians are a generalization of symplectic Grassmannians to odd-dimensional spaces. Here we compute the classical and quantum cohomology of the odd symplectic Grassmannian of lines. Although these varieties are non-homogeneous, we obtain Pieri and Giambelli formulas that are very similar to the symplectic case. We notice that their quantum cohomology is semi-simple, with enables us to check Dubrovin's conjecture for this case.

Flavia Poma
Gromov-Witten invariants in positive and mixed characteristic

I will describe how to define Gromov-Witten invariants for smooth projective schemes over any field and, more generally, over a regular scheme, focusing on the construction of a virtual fundamental class. I will show that they satisfy the fundamental axioms and some more properties (e.g. WDVV equation, reconstruction theorem). If there is time, I will compare the invariants in different characteristics for smooth projective schemes defined in mixed characteristic.

Claudiu Raicu
Affine Toric Equivalence Relations are Effective

Any map of schemes $X \to Y$ defines an equivalence relation $R = X \times_Y X \subset X \times X$, the relation of "being in the same fiber". Kollár asked whether all finite equivalence relations have this form (are effective). The answer to this question is in general negative, but is affirmative in the case of affine toric equivalence relations on affine toric varieties. I will explain the relationship between this result and the vanishing of the first cohomology group in the Amitsur complex associated to a toric map of toric algebras, and present a method for generating examples of noneffective equivalence relations.

Ronan Terpereau
Invariant Hilbert Schemes: Examples and Applications

First, we shortly remind the construction of the invariant Hilbert scheme for a reductive group G acting on a G-variety V, as introduced by Alexeev and Brion in 2005. Then, we give some of the first examples of invariant Hilbert schemes with multiplicities and present a general procedure to realise the calculation of such schemes. Finally, we show that in all our cases, the invariant Hilbert scheme gives a resolution of singularities of the quotient $V//G$.

Alan Thompson
Models for threefolds fibred by K3 surfaces of degree two

It is well known that a K3 surface of degree two can be seen as a double cover of the complex projective plane ramified over a smooth sextic curve. This talk will be concerned with finding explicit birational models for threefolds that admit fibrations by such surfaces. It will be shown that the nature of K3 surfaces of degree two allows these models to be constructed as double covers of rational surface bundles, a structure which in turn enables many of their properties to be explicitly calculated. In particular, if time permits, we will see that the degenerate fibres in these models can be explicitly studied and classified.

Michel van Garrel
Higher rank Donaldson-Thomas invariants

Donaldson-Thomas invariants of smooth complex three-folds are defined either via integration of a virtual fundamental class, or as weighted Euler characteristic. The former, earlier version works for moduli spaces of ideal sheaves (rank 1). While the latter applies to moduli spaces of sheaves of any rank, it assumes that the variety is Calabi-Yau. I will discuss elements of these constructions and talk about an attempt to generalize the first one to moduli spaces of higher rank sheaves for a class of varieties that are not Calabi-Yau.

Kiwamu Watanabe
Lengths of chains of minimal rational curves on Fano manifolds

We consider a natural question how many minimal rational curves are needed to join two general points on a Fano manifold X of Picard number 1. In particular, we study the minimal length of such chains in the cases where the dimension of X is at most 5 and the coindex of X is at most 3. As an application, we give a better bound on the degree of Fano 5-folds of Picard number 1. If we have time, I also talk about a bound of the minimal length under the mild condition on the variety of minimal rational tangents. The contents of my talk concern with my papers entitled "Lengths of chains of minimal rational curves on Fano manifolds", which was published from Journal of Algebra (Volume 325, Issue 1, 1 January 2011, Pages 163–176), and "A bound of lengths of chains of minimal rational curves on Fano manifolds of Picard number 1" (`http://arxiv.org/abs/1010.2005`).